Introduction to lithospheric geodynamic modelling

Key physical processes and concepts

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Goals of this lecture

- Present the main **physical processes and concepts** we need to consider to understand **lithospheric geodynamics**

- Provide **necessary background** for the rest of the course

Dissecting the course title

- This course is titled “Introduction to lithospheric geodynamic modelling”
    - **What does this title bring to mind for you?**

Dissecting the course title

- This course is titled "Introduction to **lithospheric** geodynamic modelling"
    - Our focus is on the **lithosphere**
        - Outermost layer of the Earth that is rigid over geological timescales
            - Thermal lithosphere: Portion of outer layers below ~1300°C
            - Crust and lithospheric mantle
    - No convecting mantle

Dissecting the course title

- This course is titled "Introduction to lithospheric **geodynamic** modelling"
  - Our focus is on **geodynamics**
    - Plate tectonics and related phenomena
    - Physical processes/topics
      - Stress and strain
      - Heat transfer
      - Deformation: Faulting and folding, rheology

Dissecting the course title

- This course is titled "Introduction to lithospheric geodynamic **modelling**"
  - Our focus is on **modelling**
    - Using computers to solve equations and simulate geodynamic processes
    - We will learn how to solve equations using numerical methods, and how to implement those numerical solutions in computer code
    - Few geodynamic processes are simple enough to be explored without the use of computers

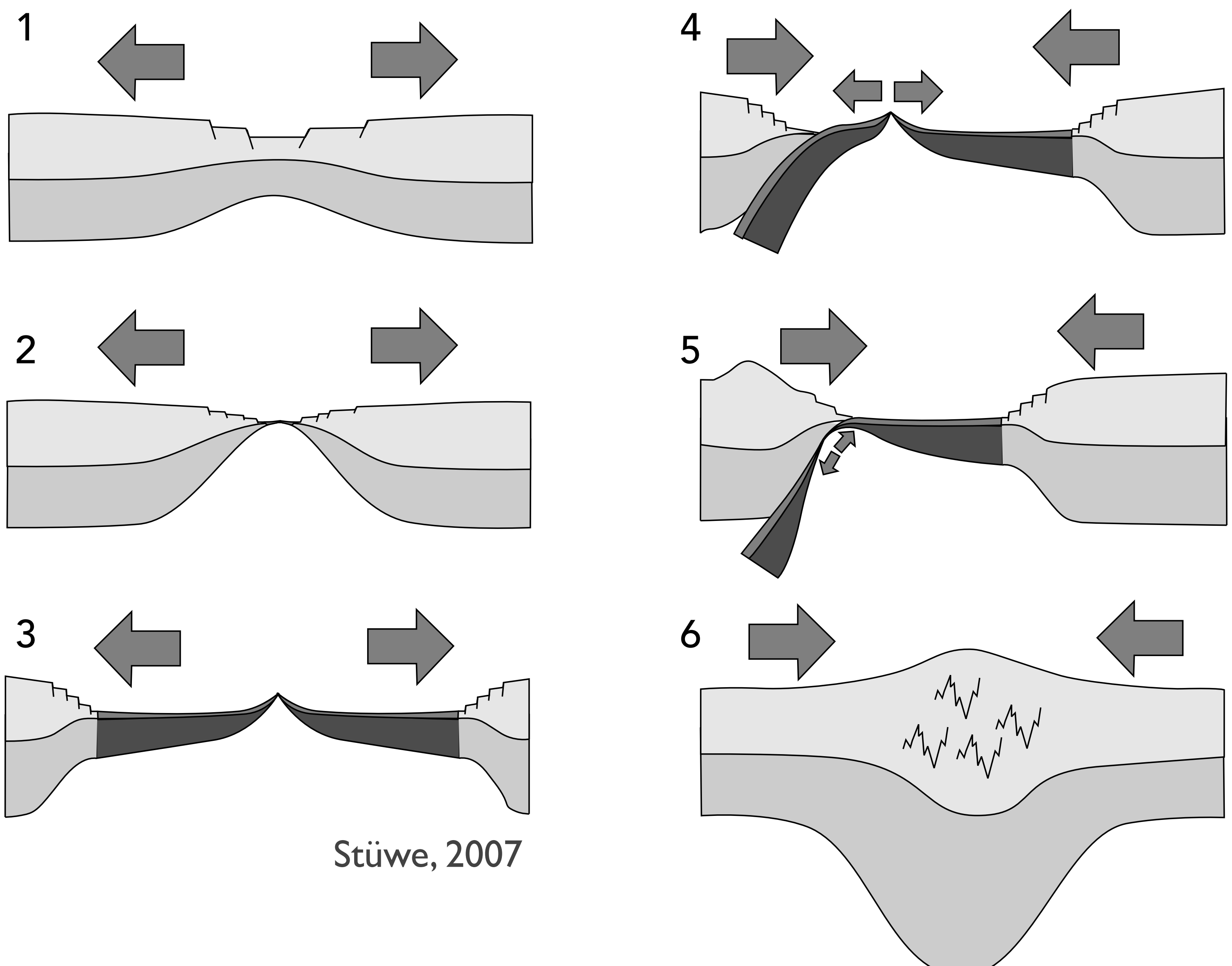
The path forward

- For the rest of this lecture, we will briefly review the different physical processes and concepts related to geodynamics
- In the following lecture we'll take some of the equations we'll see in this lecture and discuss what is needed to solve them
- In the afternoon we will review basic computing concepts by way of examples using the Python programming language
- Some of this will be review for you, but it never hurts to revisit these fundamental topics before moving on to more challenging topics

Plate Tectonics and related phenomena

Lithospheric geodynamic processes

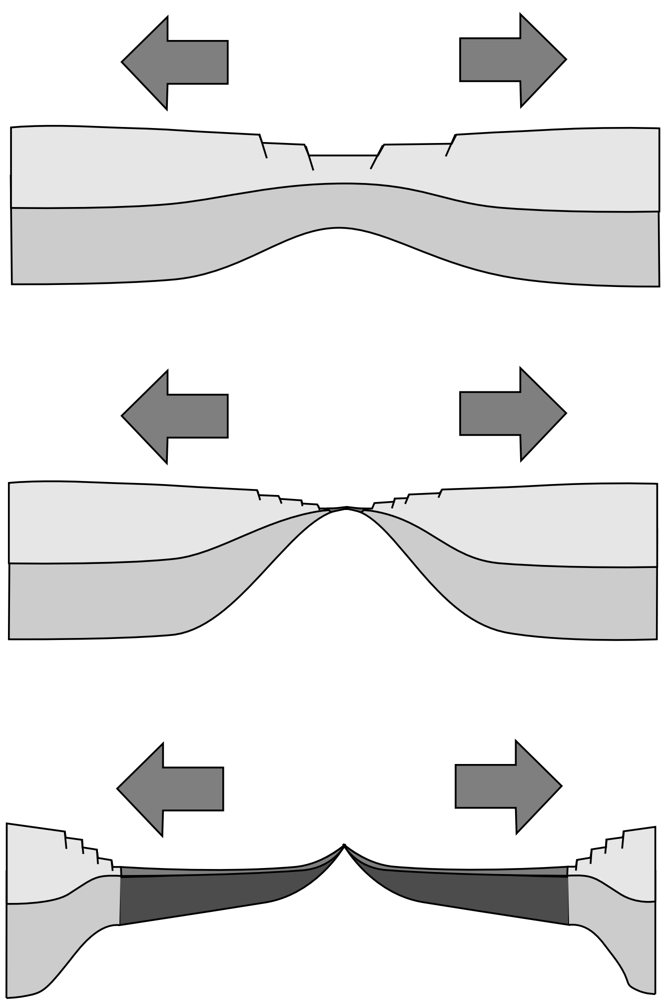
The Wilson cycle



Stüwe, 2007

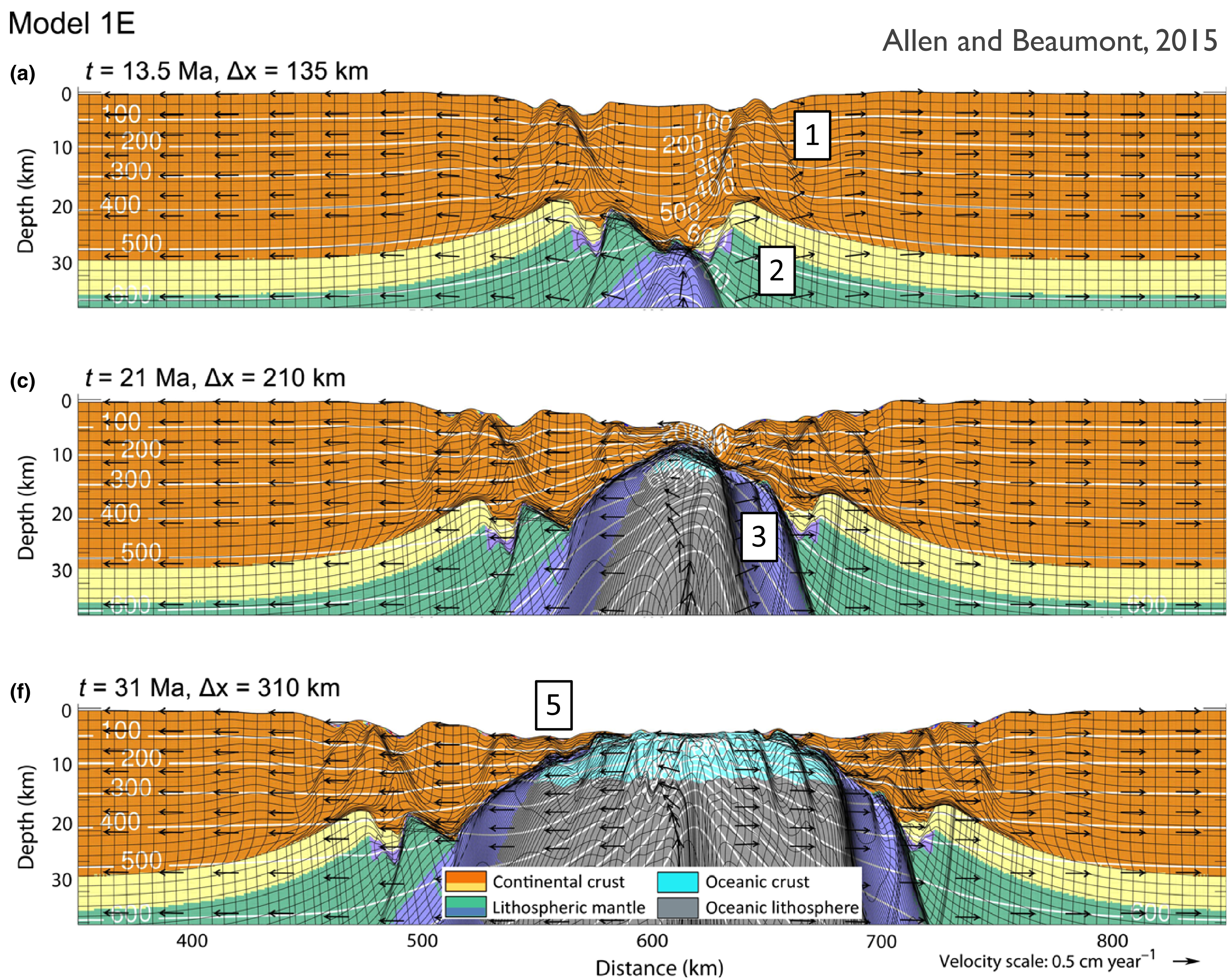
- The focus for this lecture will be on the lithosphere and the dynamic processes involved in its deformation and evolution
- Many of these processes can be directly linked to Plate Tectonics and the Wilson cycle

Rifting of the lithosphere



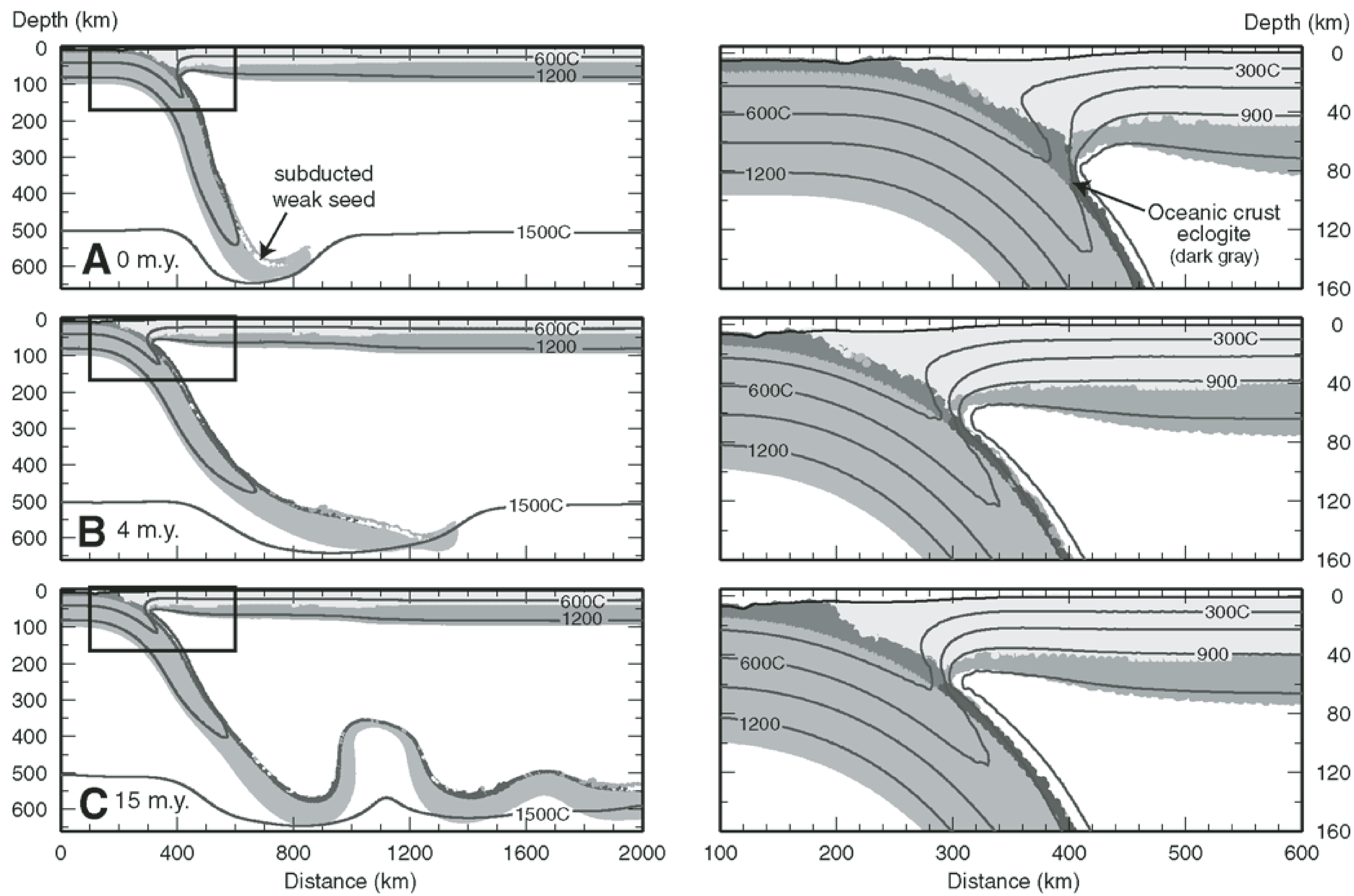
Stüwe, 2007

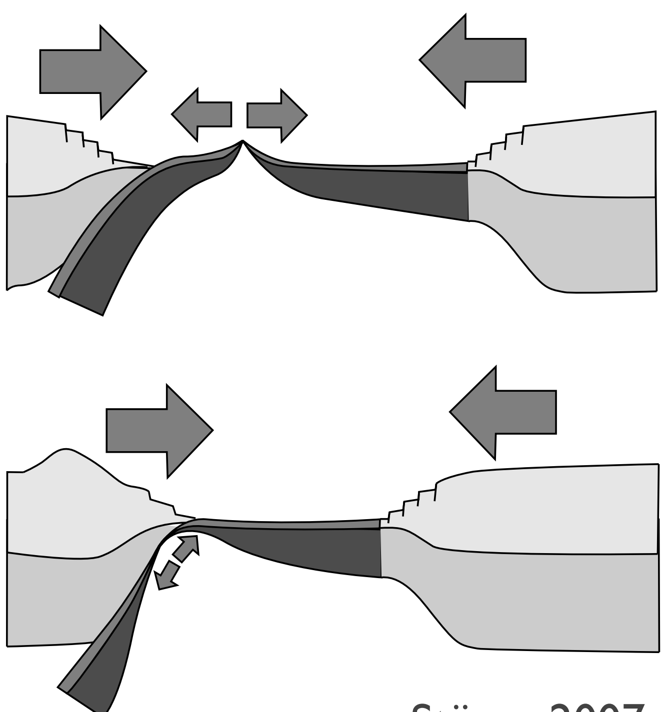
Allen and Beaumont, 2015



Oceanic subduction

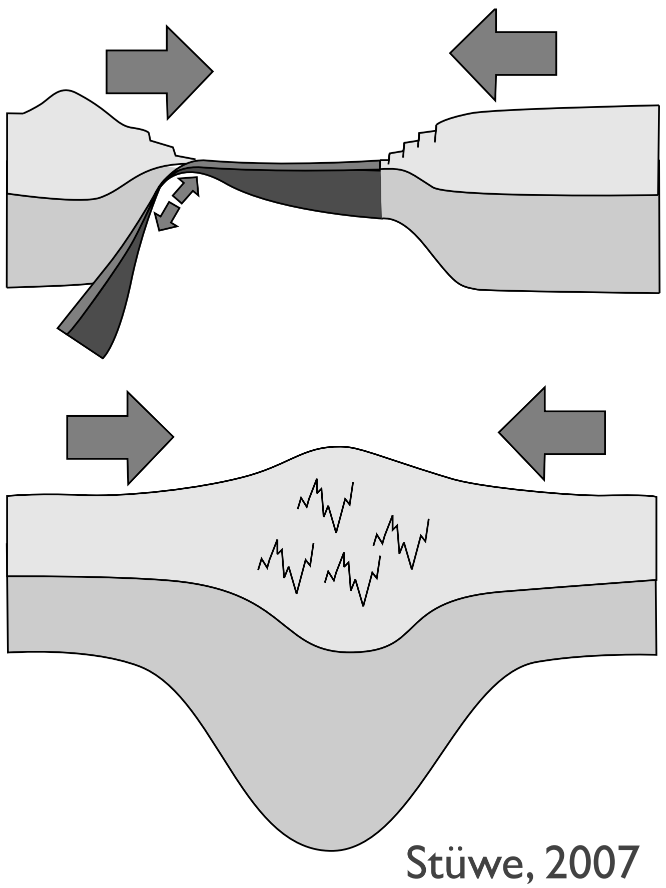
Currie et al., 2015

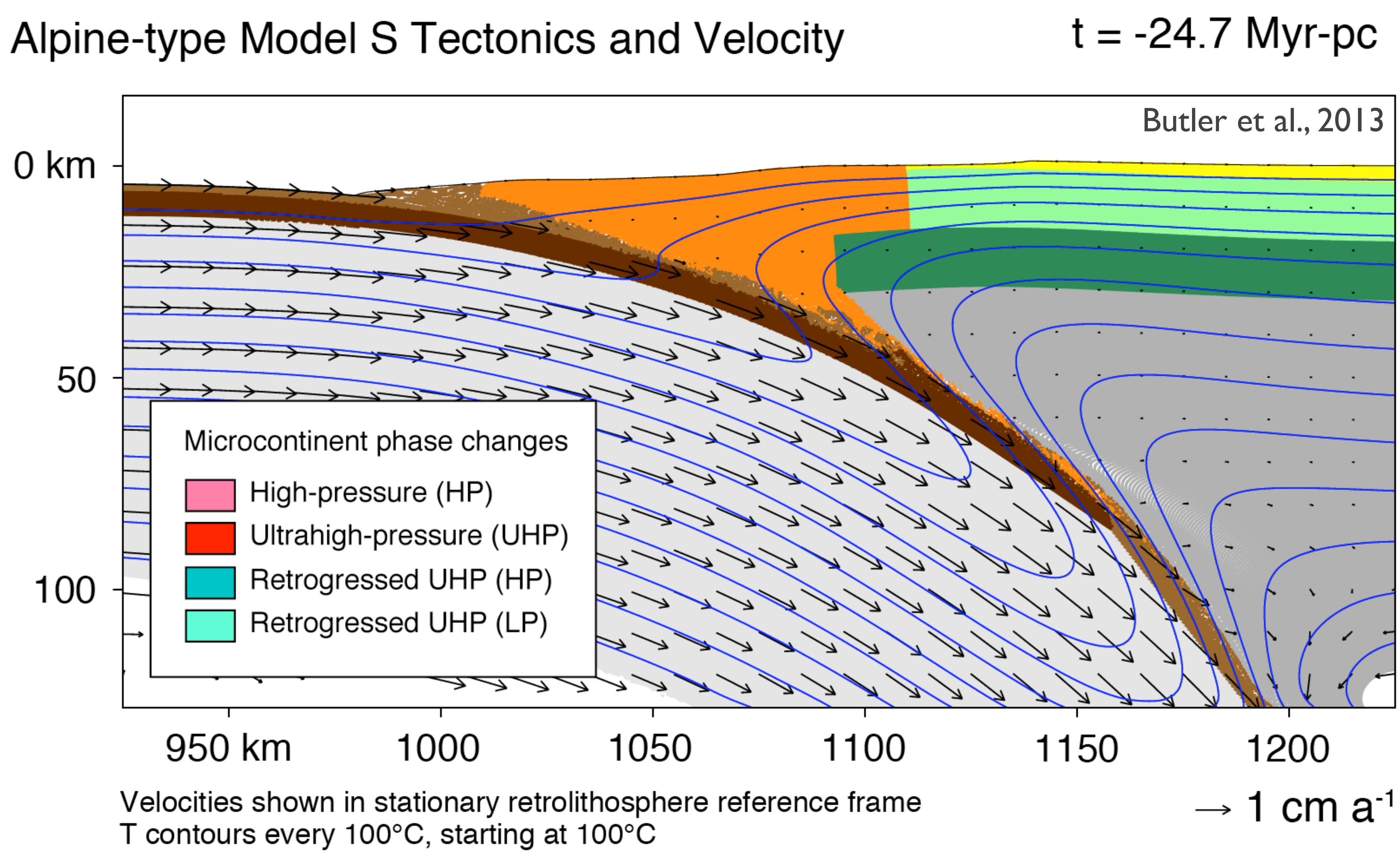




Stüwe, 2007

Continental collision





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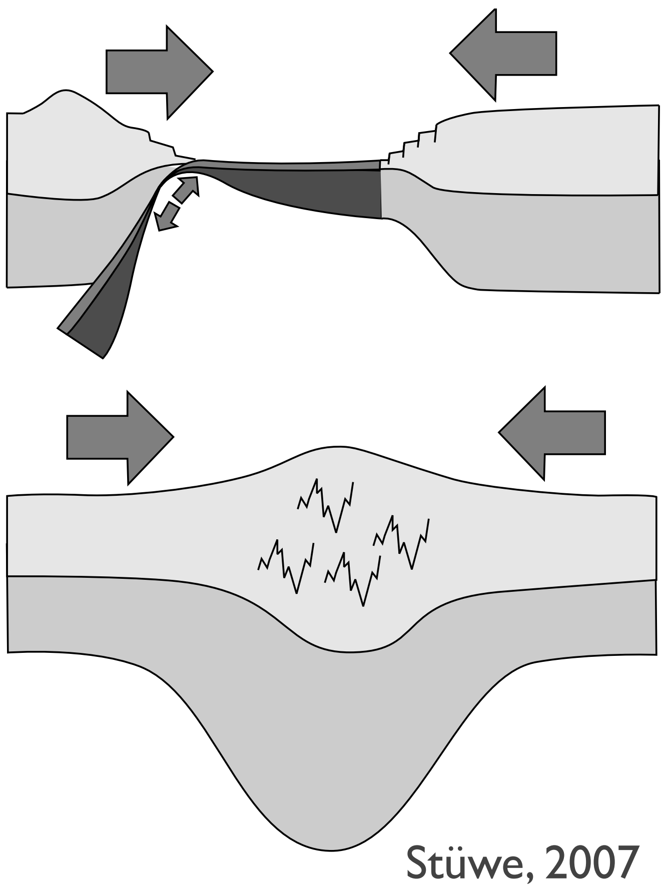
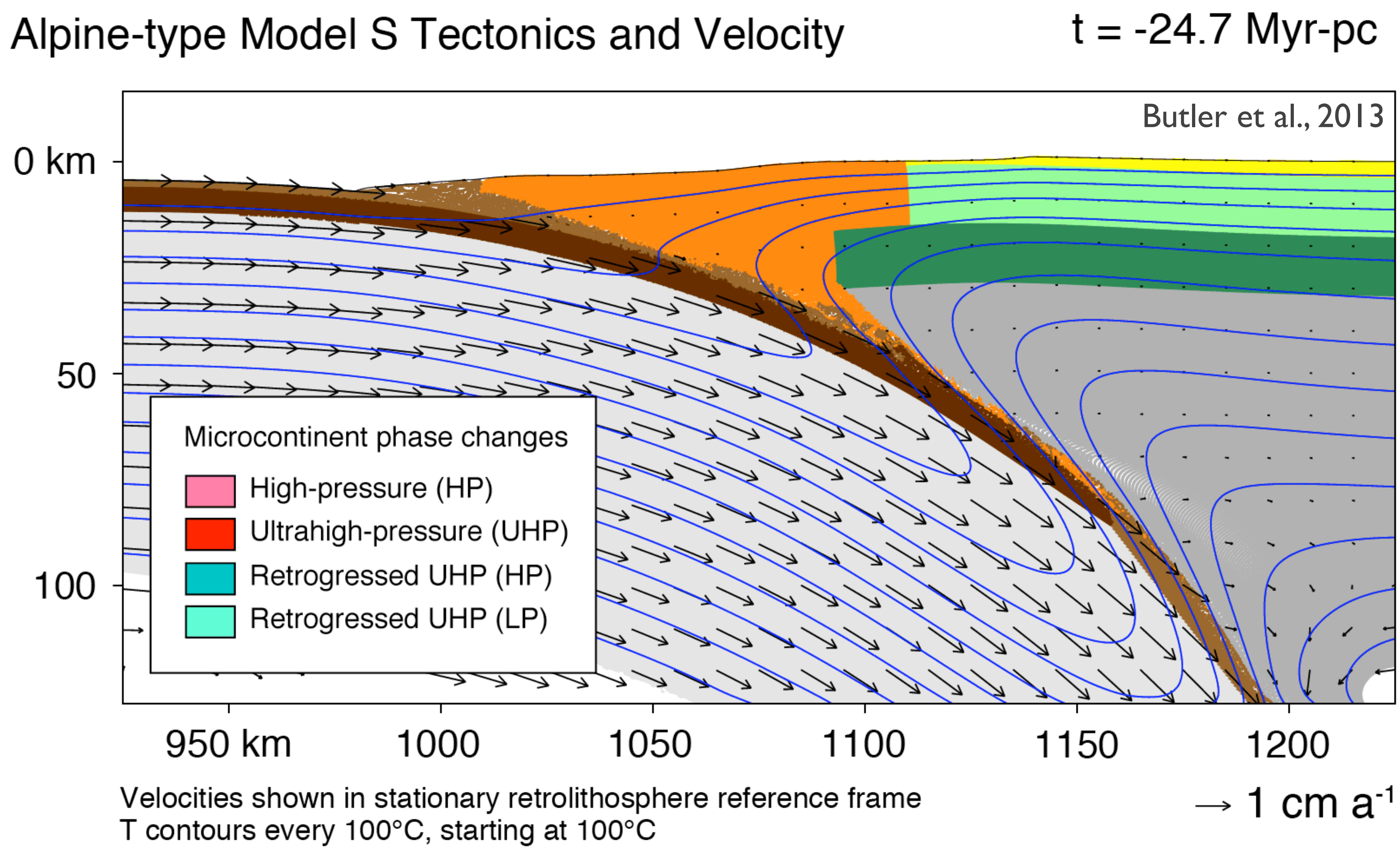
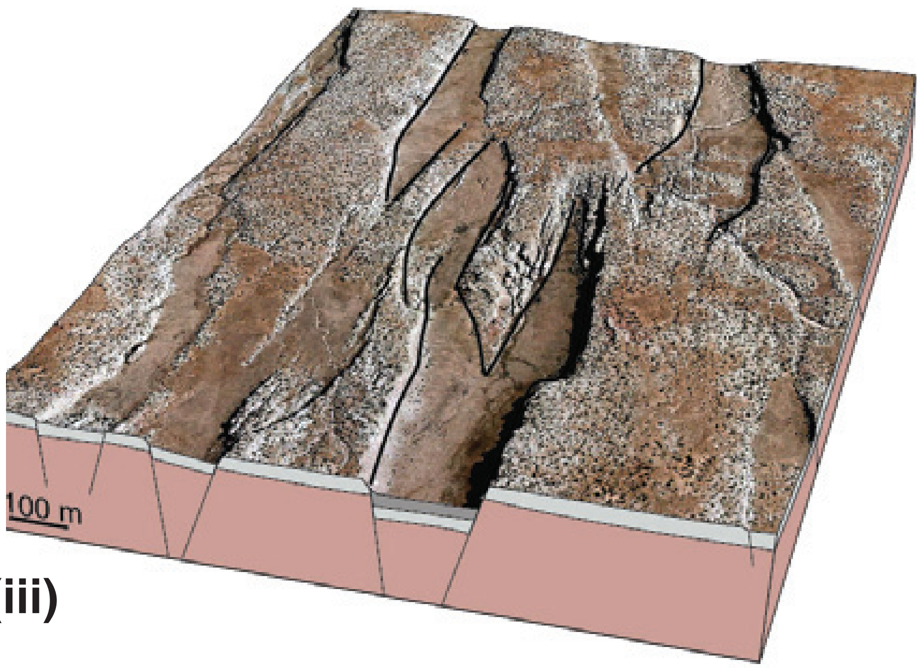




Toward three dimensions

Rifting



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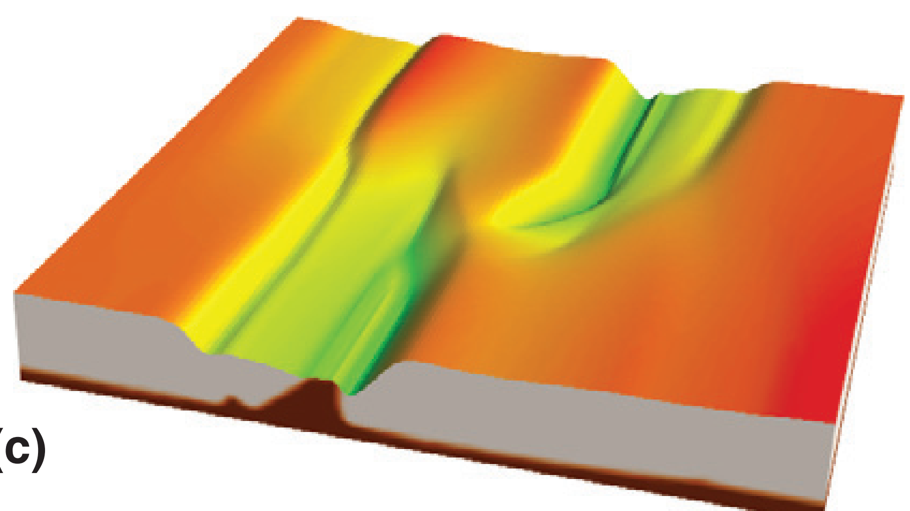

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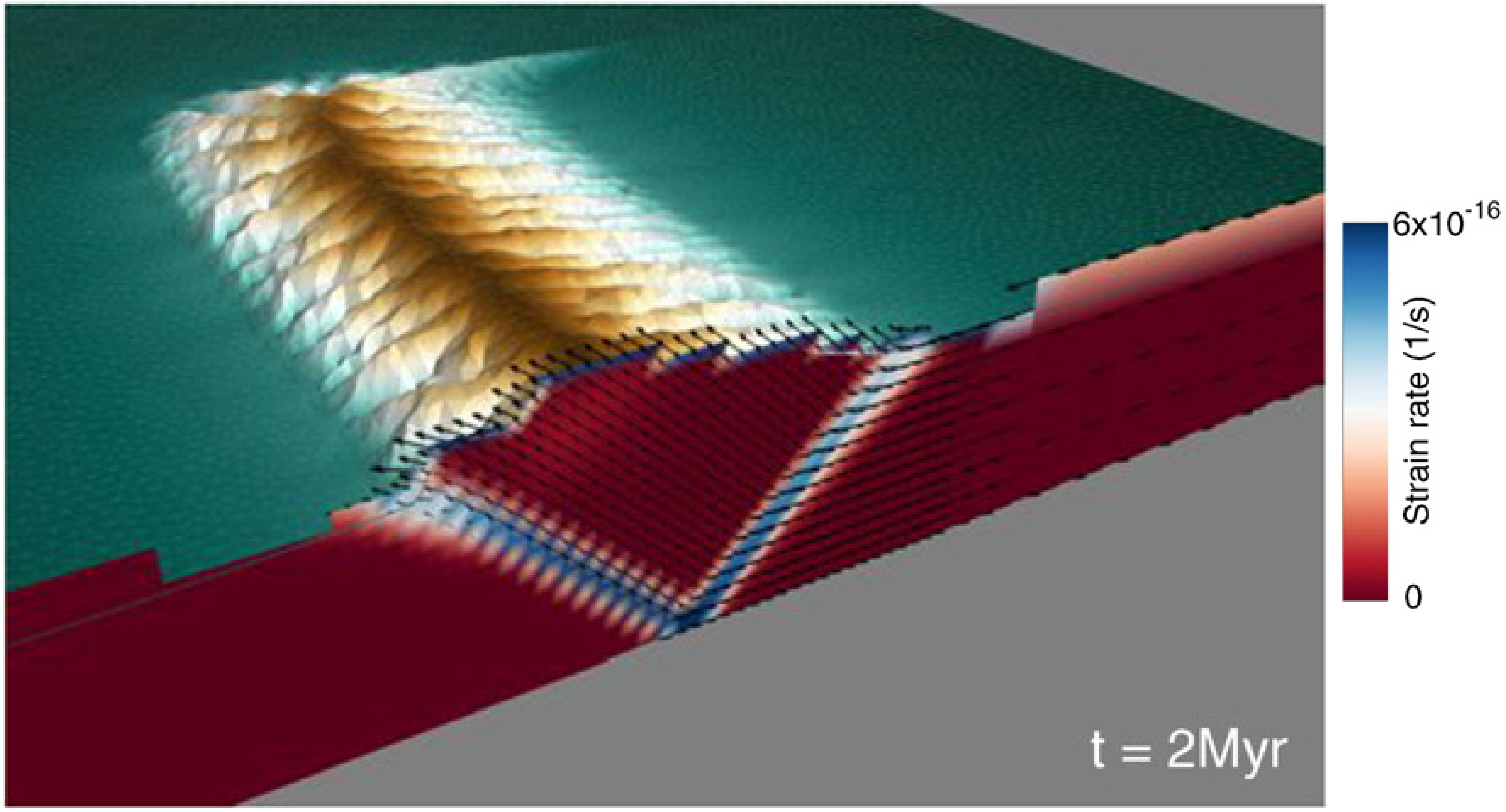
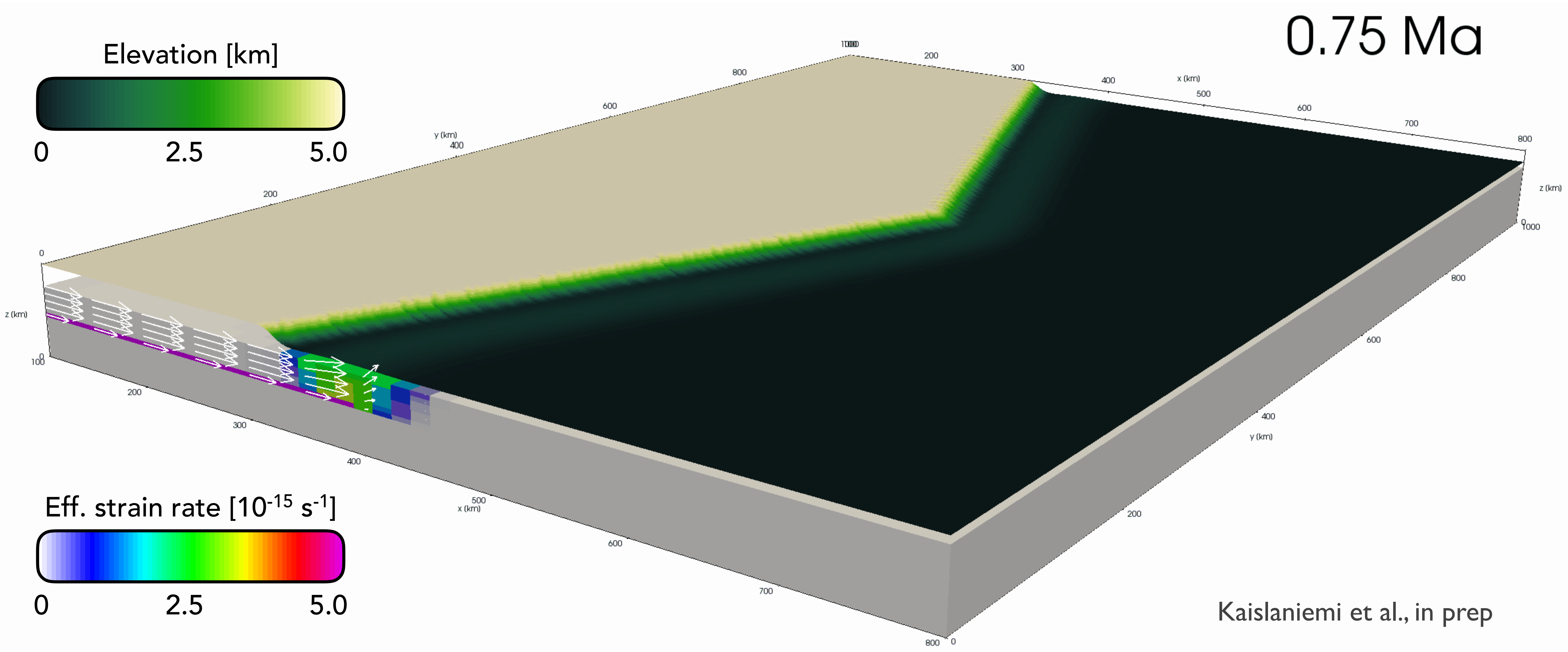
Allken et al., 2012

Continental collision



Braun and Yamato, 2010

Fold-and-thrust belt growth and erosion



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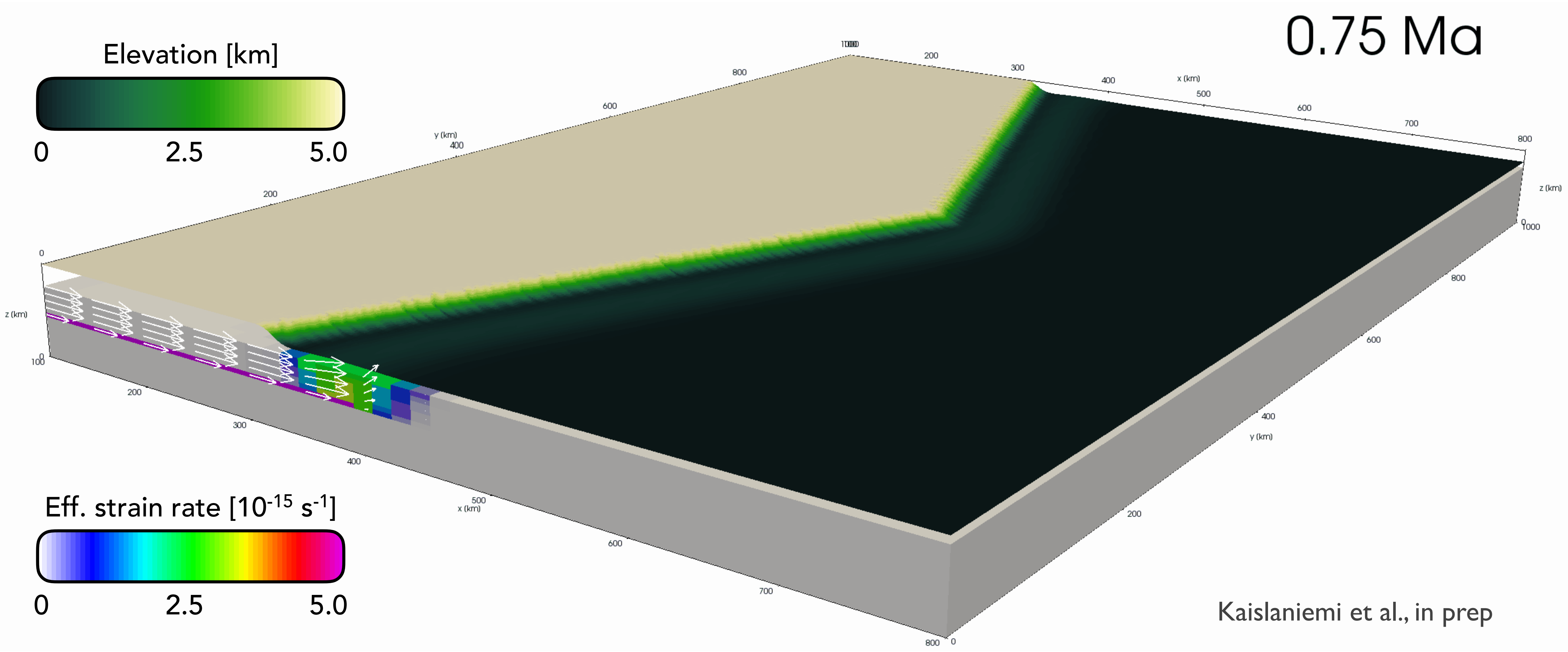


Kaislaniemi et al., in prep

What is a model?

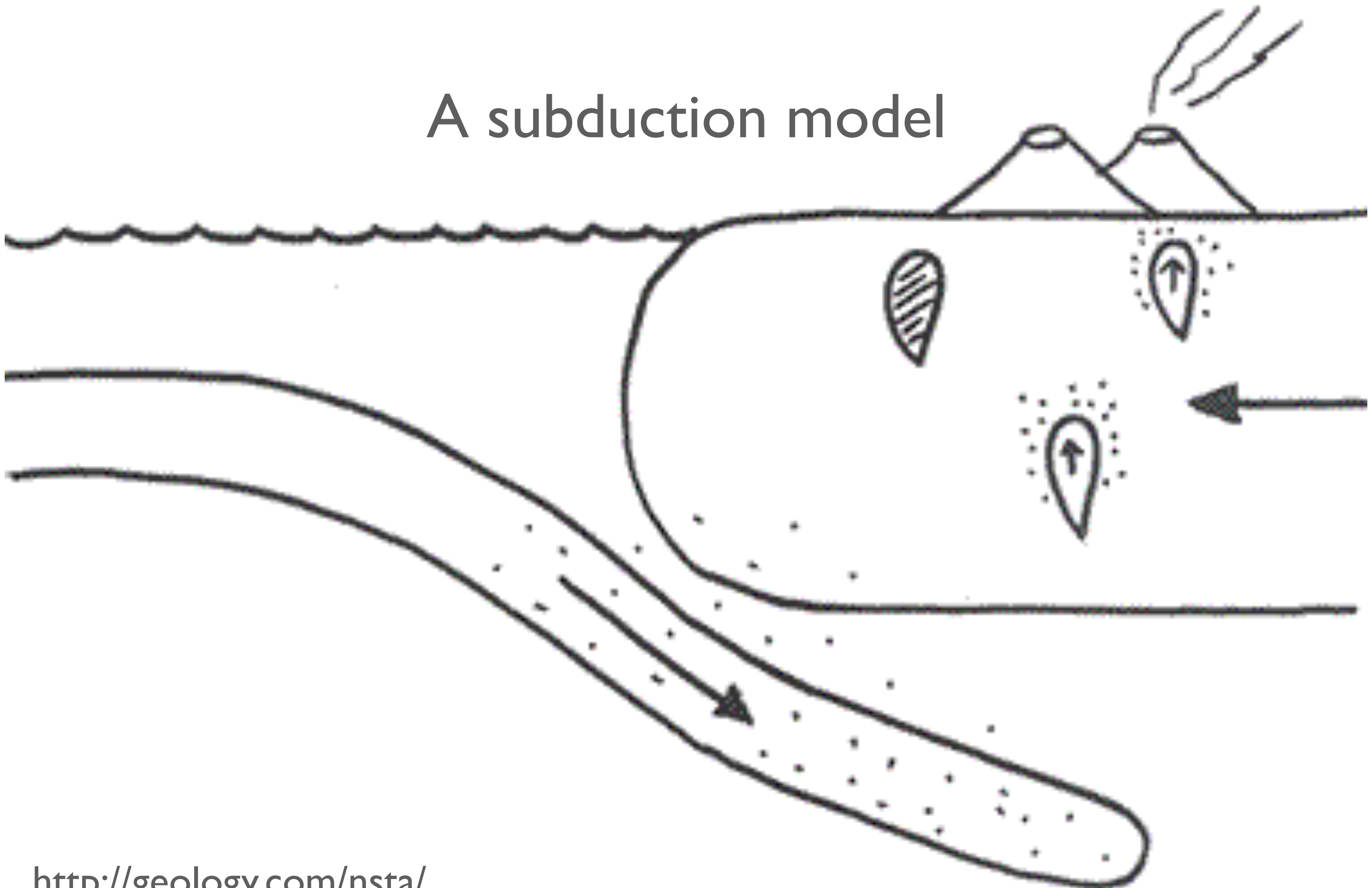
What is a model?

- "A **model** it tool used to describe the world around us in a *simplified way* so that we can *understand it better*"

  Stüwe, 2007

What is a model?

A subduction model



http://geology.com/nsta/

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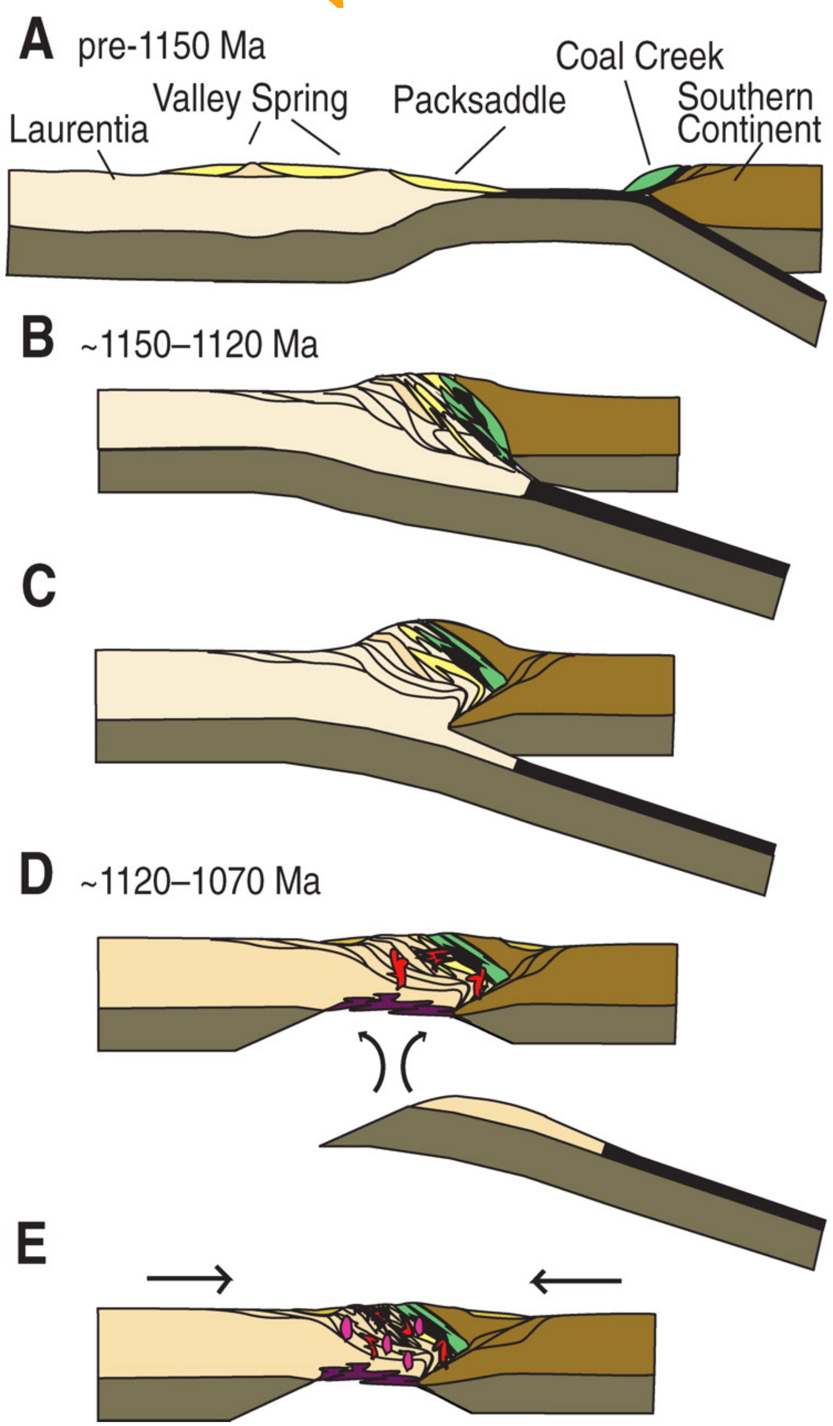
Types of geological models



Mosher et al., 2008

- **Tectonic diagrams** are a familiar form of model to help clarify the time evolution of a study area
  - Typically this kind of model is used to simplify the complex modern geology and restore it to a pre-deformation state
  - These models, though, have no basis in physics

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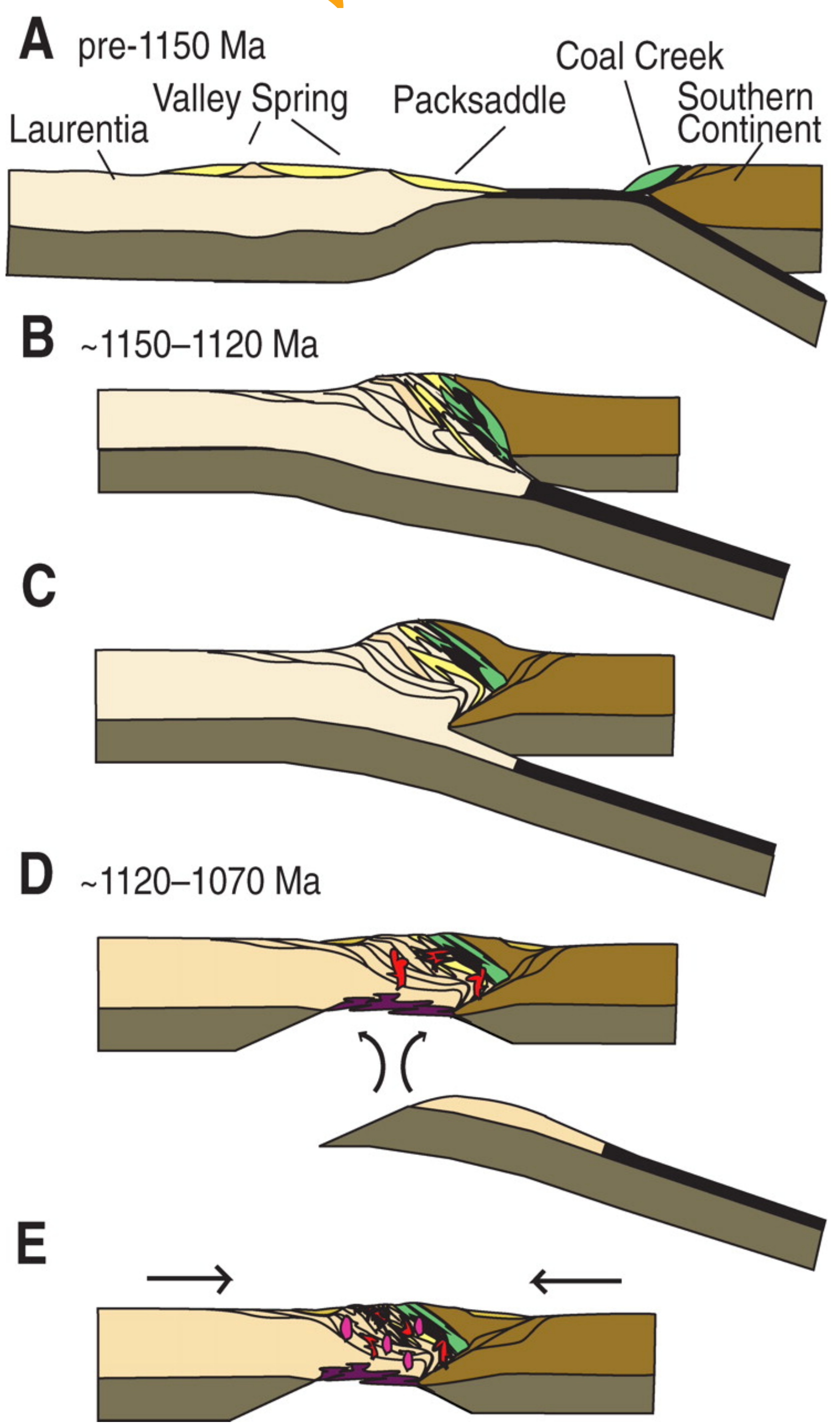

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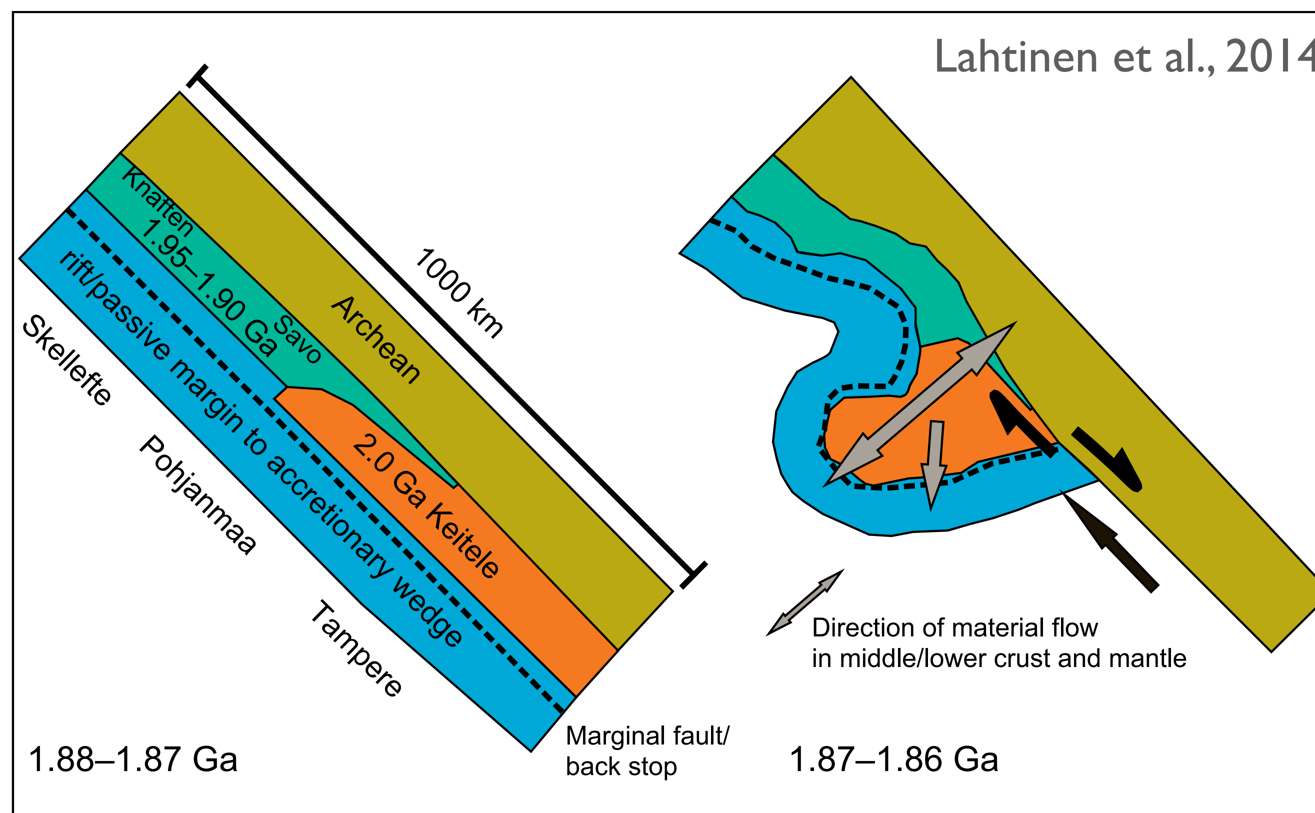

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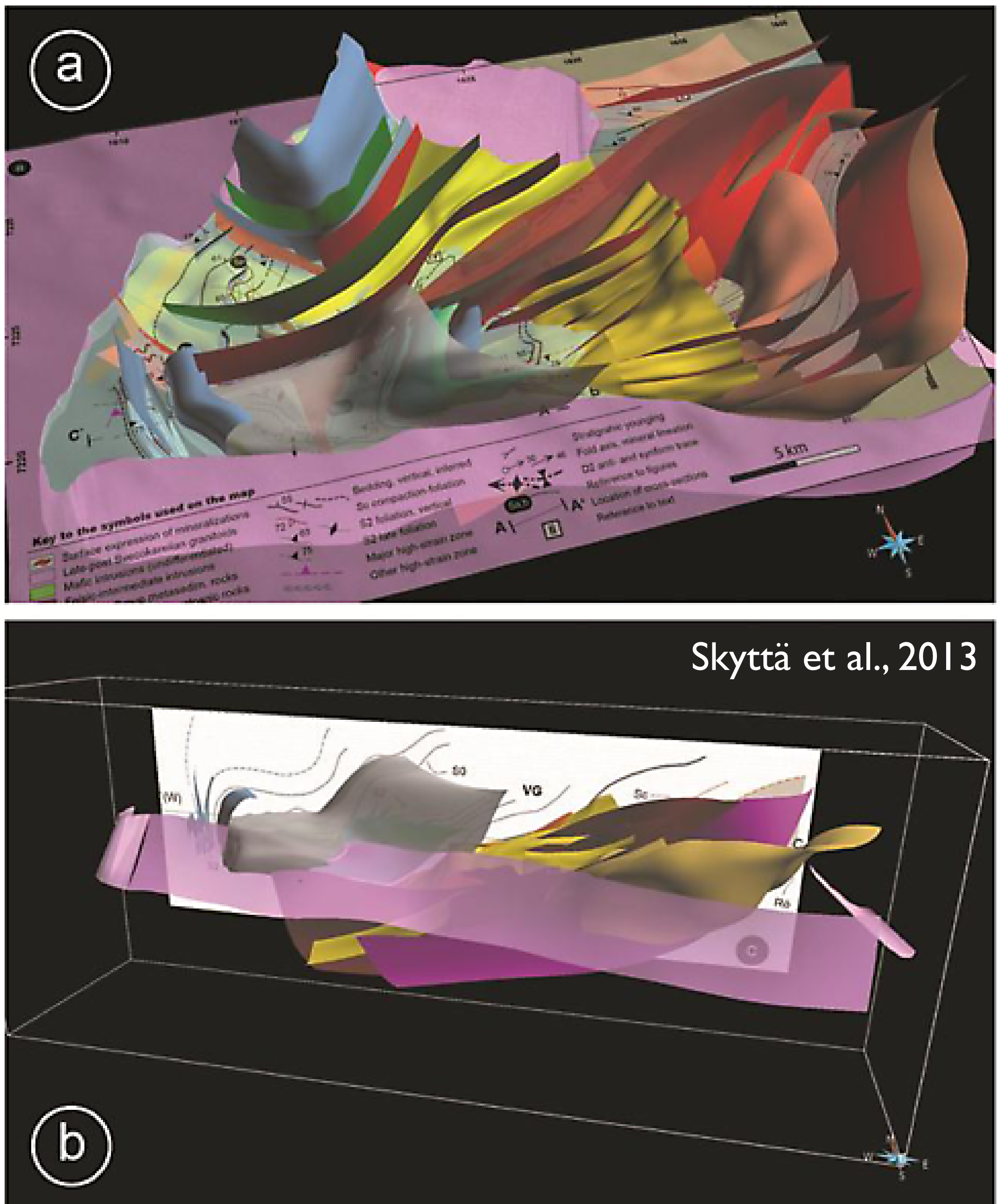
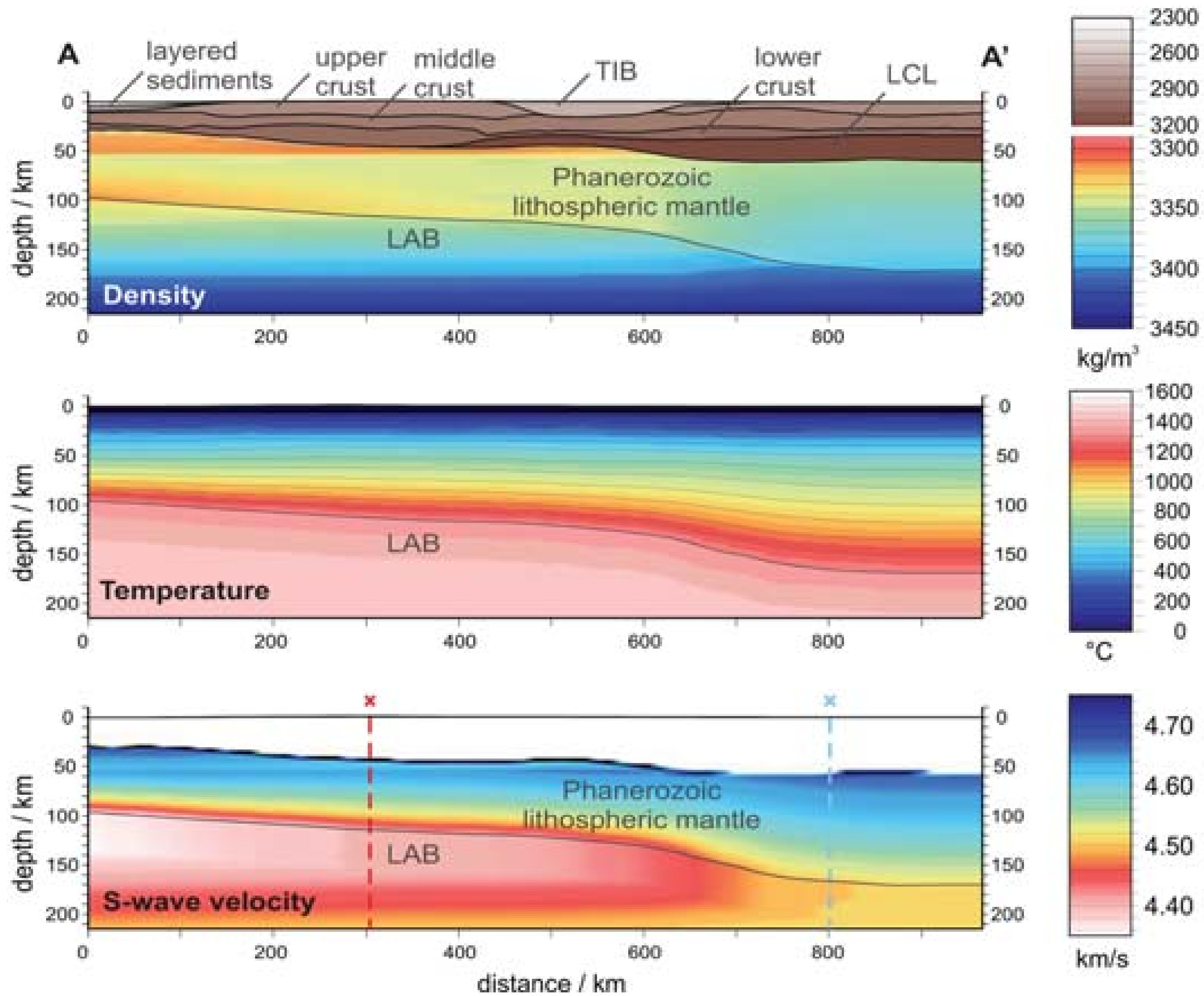


- **3D structural** or **geological models** are closer to reality in that they are based on a combination of surface and subsurface geological and geophysical observations
- The primary goal of these models is data visualisation, again helping us understand complex geometries
- Models of this type typically do not simulate physical processes

Integrated geophysical modelling

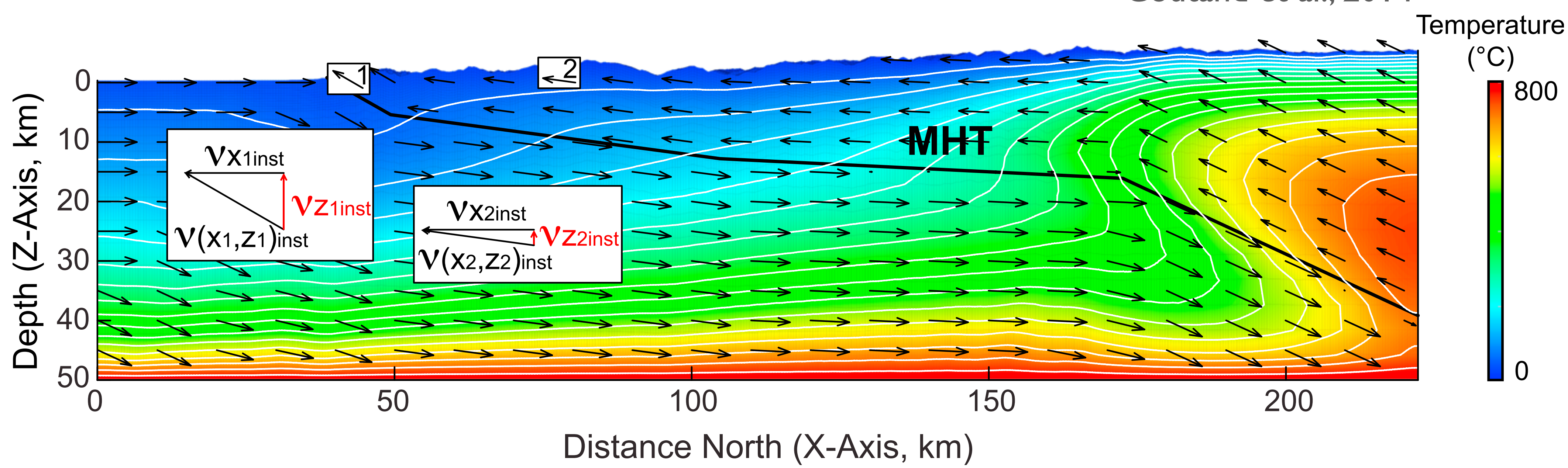


Gradmann et al., 2013

- **Integrated geophyiscal models** use a combination of an input crustal structure and composition, and rock thermal properties to calculate various properties of the lithosphere (gravity anomalies, seismic velocities, surface heat flow, etc.)
- These models involve a 2D or 3D geometrical model and calculation of heat transfer in the lithosphere and upper mantle

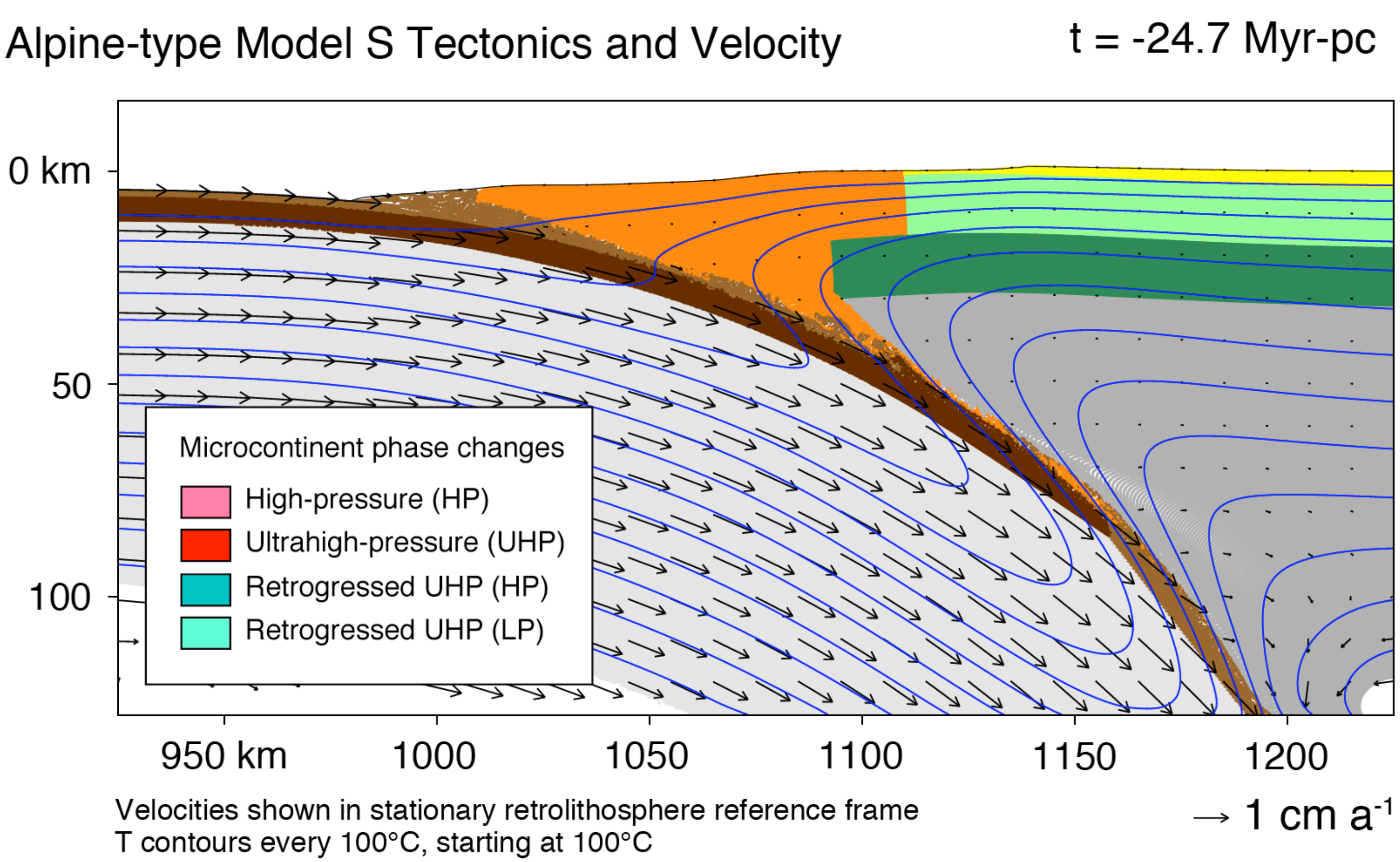
Types of geological models

Coutand et al., 2014



- **Thermo-kinematic** (or **thermokinematic**) **models** simulate both mass transport and heat transfer using a pre-defined velocity field and input rock thermal/physical properties
- Models of this type can be compared to a number of observables, including surface heat flow and mineral cooling ages, and typically have a geometry based on surface geological observations and geophysical data such as reflection seismics

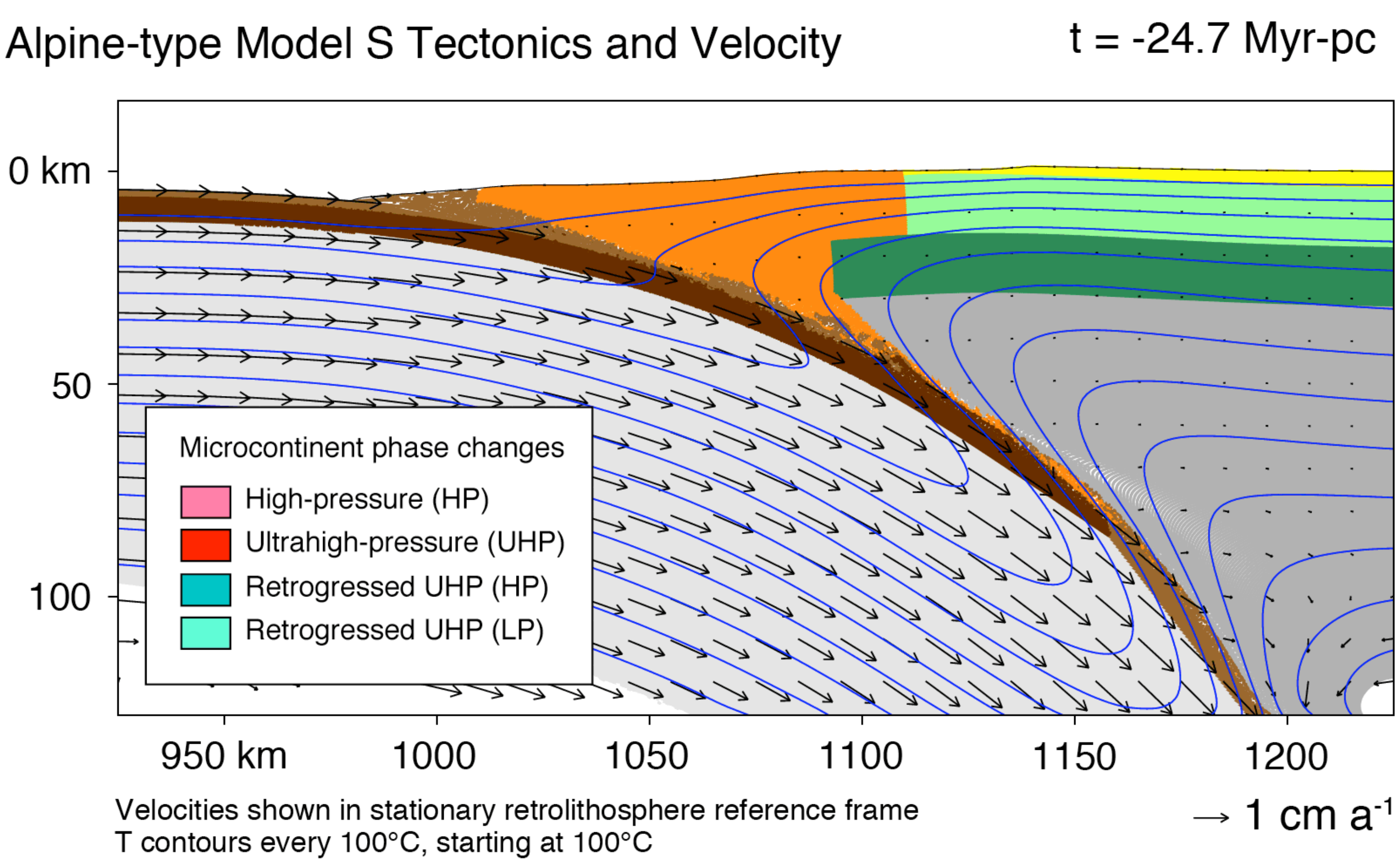
Types of geological models



Butler et al., 2013

- **Thermo-mechanical models** truly simulate lithospheric dynamics
  - Internal deformation in the model is determined based on the material properties of rock in the model and not prescribed
  - Heat transfer will vary as a result of model deformation, but also affect the model material properties

Types of geological models



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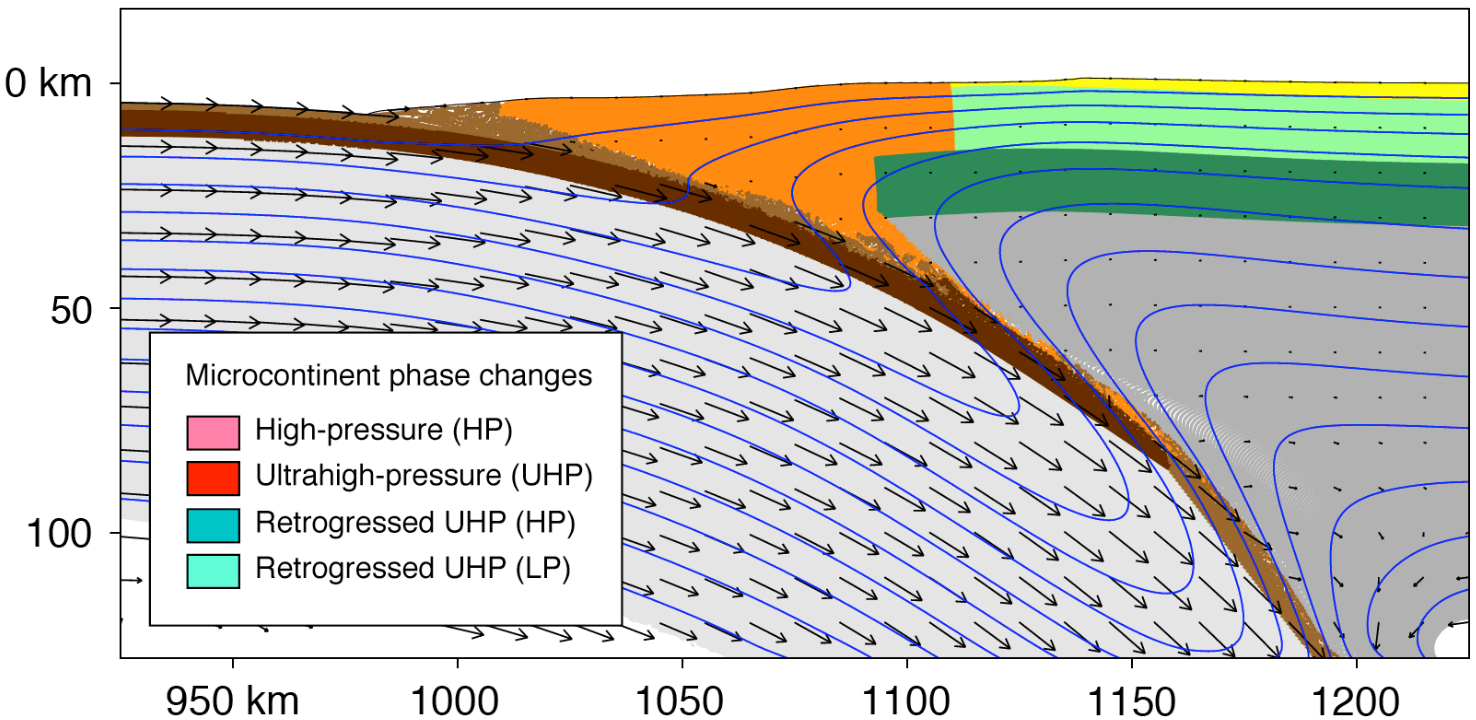
Types of geological models

Alpine-type Model S Tectonics and Velocity t = -24.7 Myr-pc



Velocities shown in stationary retrolithosphere reference frame
T contours every 100°C, starting at 100°C
→ 1 cm a^{-1}

Butler et al., 2013

- **Thermo-mechanical models** truly simulate lithospheric dynamics
  - This type of model offers the greatest predictive power, but can be difficult to directly link to geological observations because the model evolution is not known *a priori*
  - This kind of model is the focus for the remainder of this presentation

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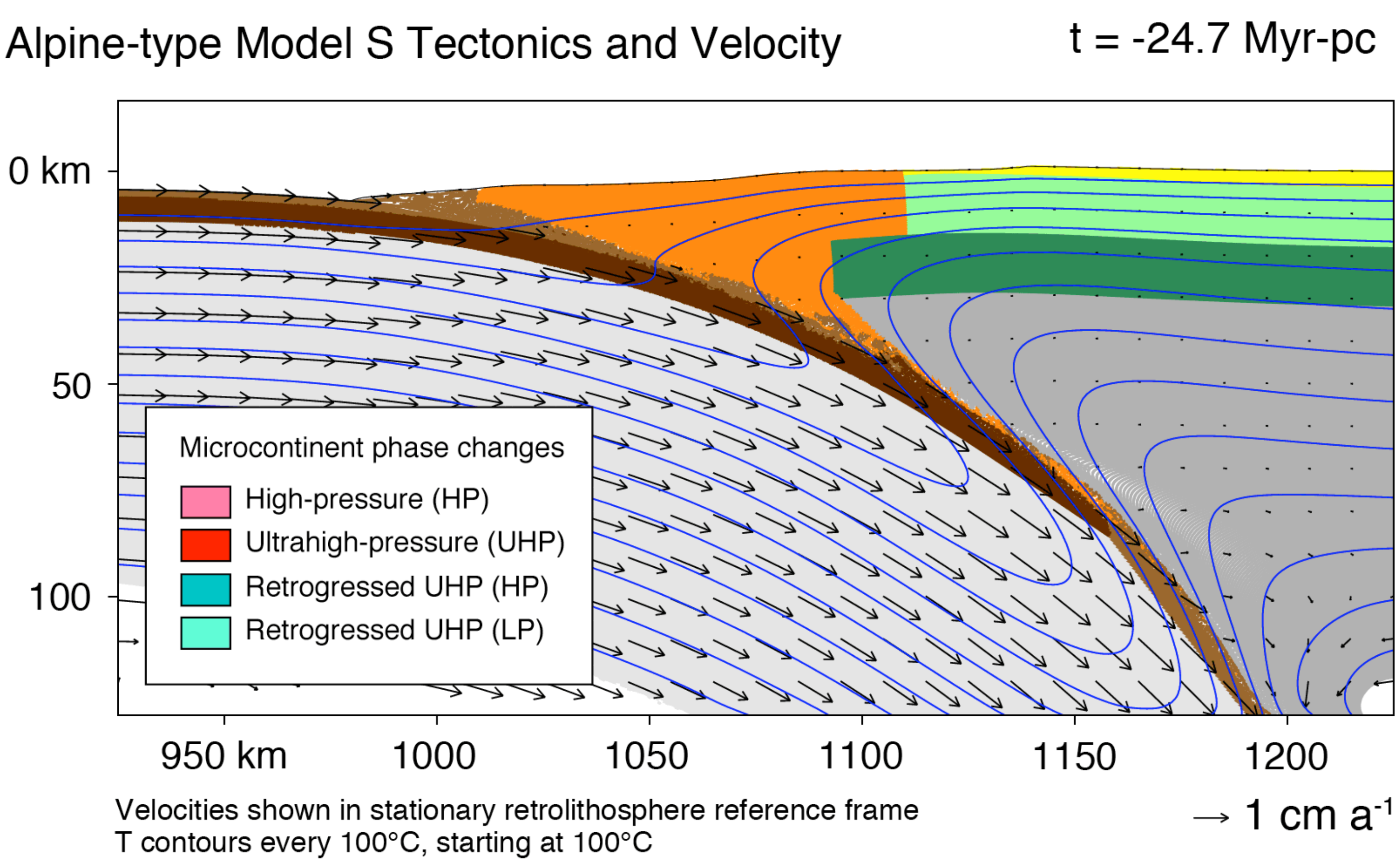

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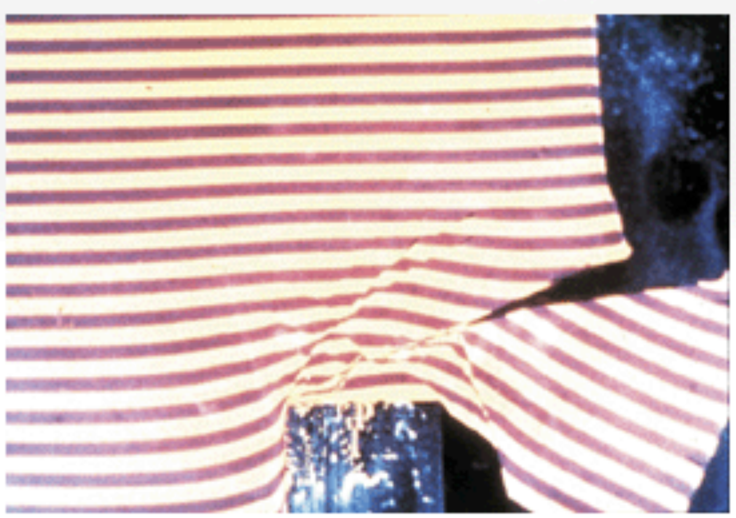
Analogue versus numerical models

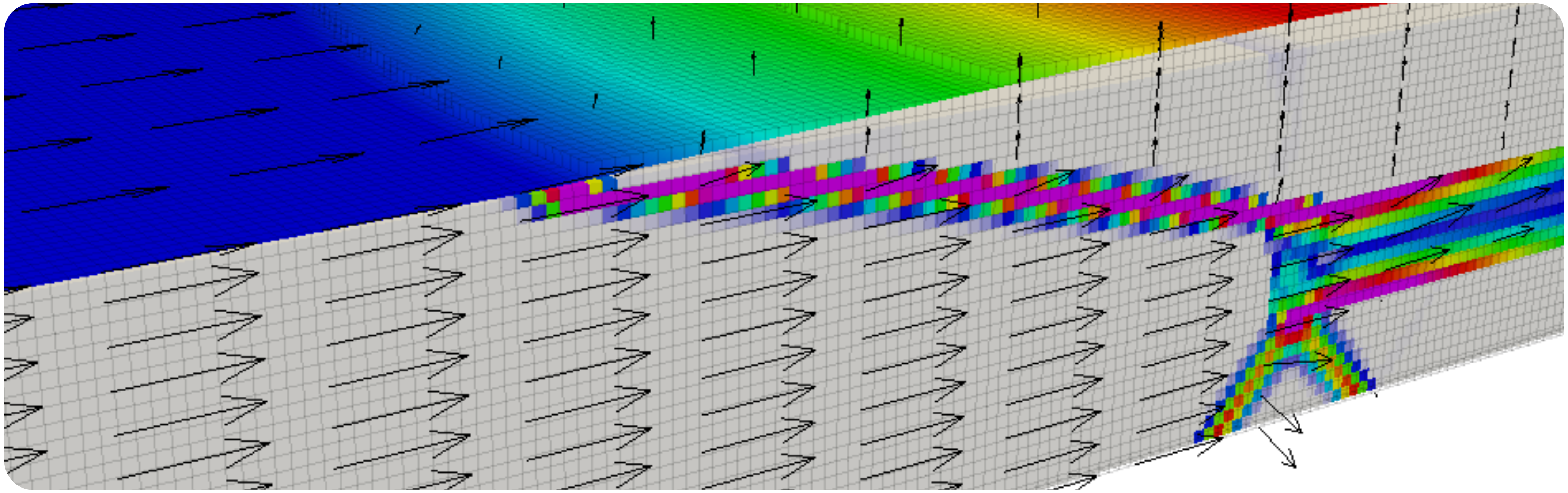


Tapponnier et al., 1982

- **Analogue models** are an alternative to thermomechanical models where materials analogous to Earth materials are used to simulate deformation of the Earth in physical models
  - These models do not prescribe any material behavior, but rather allow the material to deform subject to imposed deformation at the boundaries
  - Though these are a viable alternative to numerical models, it is difficult to simulate temperature-dependent materials and scaling of the model properties can be a problem

Physical model concepts: Earth as a continuum

What does that even mean?



Velocities and strain rates in a lithospheric geodynamic model

- Most geodynamic models treat the Earth as a **continuum** such that there are no material gaps or voids at the macroscopic scale
    - Field variables such as pressure, velocity, or stress are thus fully continuous
- In this context the Earth is a fluid with a very high viscosity (typically 10^{18} - 10^{23} Pa s)

Earth as a fluid? Even the lithosphere?

- **Fluid**: Any material that flows in response to an applied stress

- Differences between **solids** and **fluids**

Solids	Fluids
Strain from being stressed	**Continuous** deformation under applied forces
Stresses related to strains	Stresses related to **rates of strain**
Strain result of displacement gradients	Strain result of **velocity gradients**

- **Rheological** (or **constitutive**) **law**: An equation relating stress to strain rates in a fluid

Fluid mechanics

- **Fluid mechanics** is the science of fluid motion

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- Based on conservation of three basic physical property and their corresponding mathematical representations
  - **Conservation of mass** - The continuity equation
  - **Conservation of momentum** - The momentum equation
  - **Conservation of energy** - The heat transfer equation

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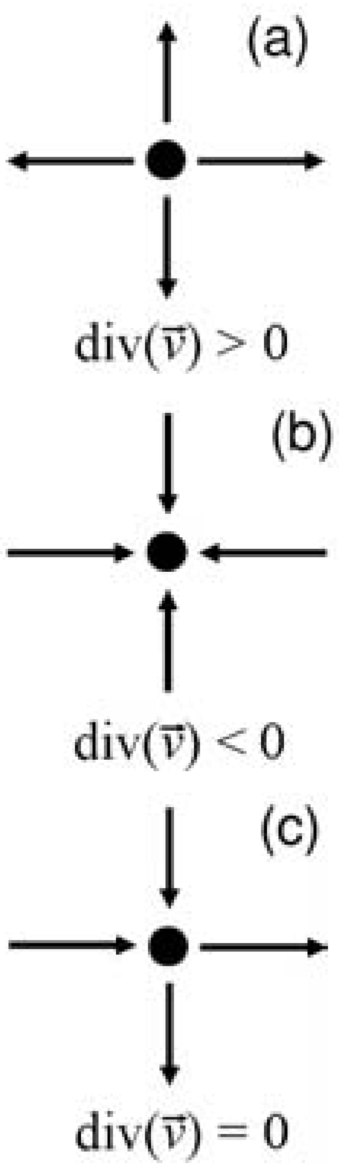
The continuity equation



Gerya, 2010

- Calculations in the continuum are performed by considering an infinitesimal volume of the material, the local volume
- The general form of **conservation of mass** for a local volume of a continuum in an Eulerian reference frame is

$$\frac{\partial \rho}{\partial t} + \rho \left(\nabla \cdot \boldsymbol{V} \right) = 0$$

Change in local density

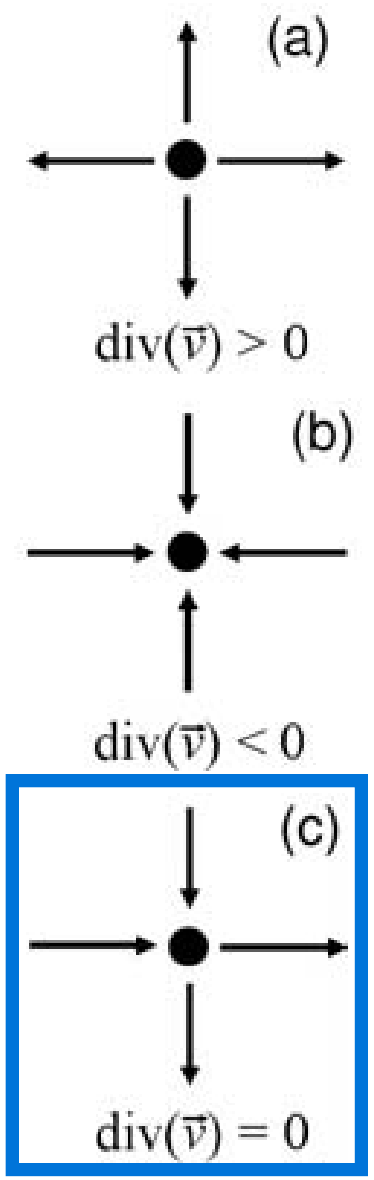
Mass or volume flux (divergence of velocity)

where ρ is the local density, t is time and $\boldsymbol{V}$ is the local velocity

$$\frac{\partial \rho}{\partial t} = -\rho \left(\nabla \cdot \boldsymbol{V} \right)$$

Alternative form

The continuity equation



Gerya, 2010

- It is common in geodynamic numerical models, particularly in the crust or lithosphere, to assume the material is incompressible
- In this case, the **continuity equation** simplifies to

$$\nabla \cdot V = 0$$

stating simply that there is no divergence in the velocity field of the continuum

- In many numerical models, this condition is not strictly obeyed, allowing a very small amount of compressibility in the materials

What drives a fluid to flow?

Sir George Stokes

- At this point, we have established that geodynamicists often model the Earth as a continuous, highly viscous fluid. Since we're interested in the dynamics of this fluid, a reasonable question to ask is **what might cause a fluid to flow?**

The momentum equation

Sir George Stokes

- The basic relationship that thus determines the dynamics of material in the continuum is conservation of momentum, the balance of internal and external forces acting on the material

- The **conservation of momentum** for a fluid subject to gravity is the Navier-Stokes equation

$$\nabla \cdot \eta(\nabla \boldsymbol{V} + \nabla \boldsymbol{V}^{\mathrm{T}}) - \nabla P - \rho g = \rho \dot{\boldsymbol{V}}$$

Fluid velocity | Fluid pressure | Body forces | Acceleration

where η is the fluid shear viscosity, P is pressure, g is the acceleration due to gravity, and $\dot{V}$ is the material time derivative of the fluid velocity (acceleration)

The momentum equation

Sir George Stokes

- For highly viscous fluids with a very small Reynolds number the acceleration term of the Navier-Stokes equation can be ignored reducing to the equation of **Stokes flow** (and simplifying the solutions)

$$\nabla \cdot \eta(\nabla \boldsymbol{V} + \nabla \boldsymbol{V}^{\mathrm{T}}) - \nabla P = \rho g$$

Fluid velocity — Fluid pressure — Body forces

- It is trivial to demonstrate that the Reynolds number of most geodynamic flows is extremely low

$$\mathbf{Re} = \frac{\rho \boldsymbol{V} L}{\eta} \qquad \frac{\text{Inertial forces}}{\text{Viscous forces}}$$

The Reynolds number

Pre-coffee exercise

- The Reynolds number determines whether or not a fluid flow should be expected to be turbulent or laminar
  - The critical value of the Reynolds number is ~2000, above which flow is turbulent

$$\mathrm{Re} = \frac{\rho \boldsymbol{V} L}{\eta}$$

The Reynolds number

- Using your best guess at the equation values, **estimate the Reynolds number for convection of the upper mantle**
  - Do we need to worry about turbulence?

Physical model concepts: Stress and strain

Forces

- **Force**: A push or pull applied to a body.
  Force = mass x acceleration (Newton's second law)
- **Units**: Newtons [N]; $1\ N = 1\ kg\ m\ s^{-2}$

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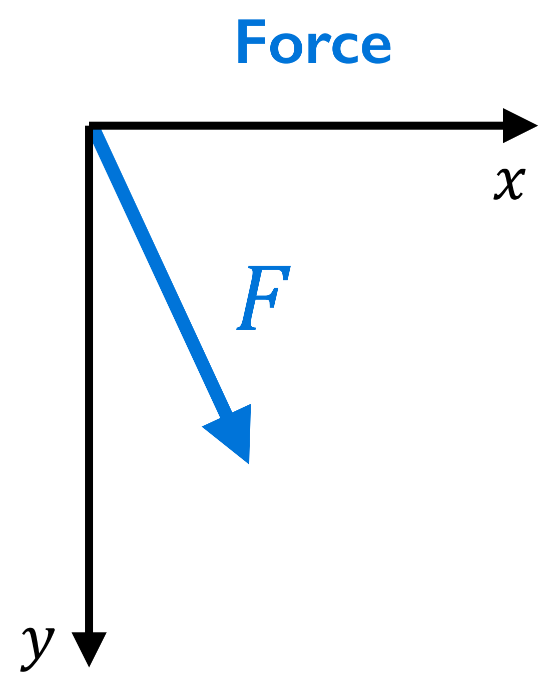
- **Representation**: Vector
- **Example**: Gravity (directed toward center of Earth)



after Twiss and Moores, 2006

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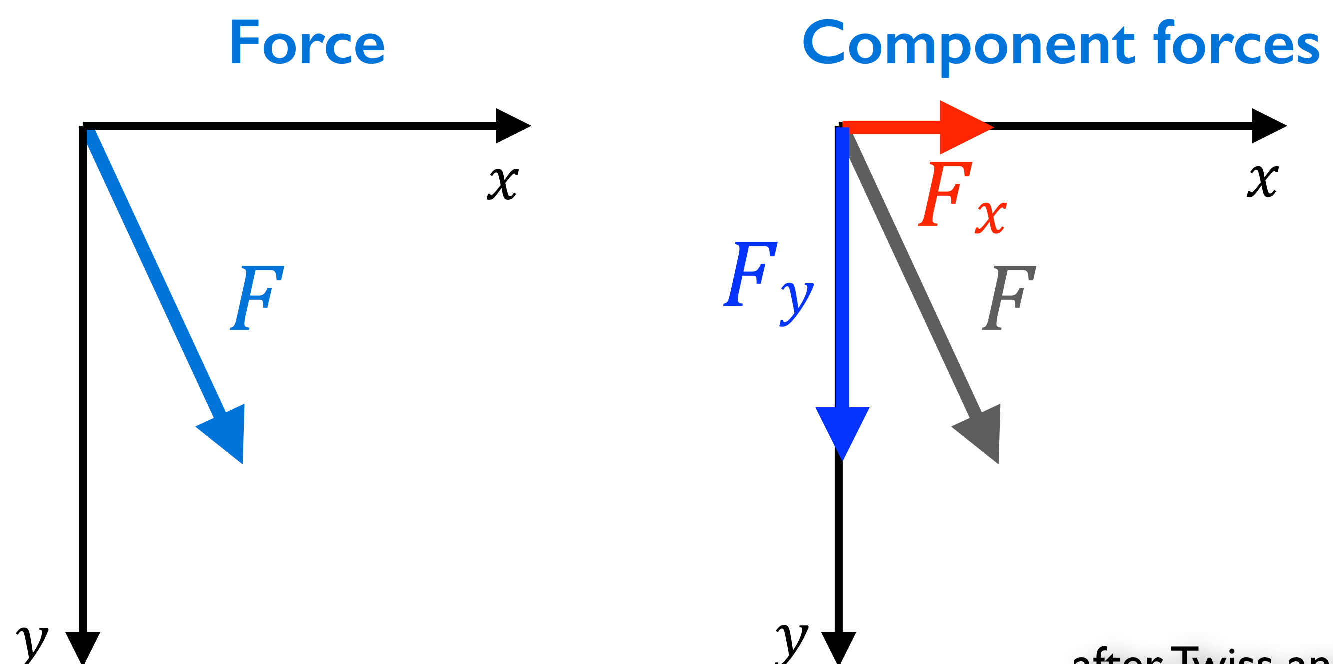

after Twiss and Moores, 2006

Body forces versus surface forces

- **Body force**: Forces that act throughout the volume of a solid. Proportional to its volume or mass.
  - Example: Slab pull (gravity)

- **Surface force**: Forces that act on the surface area bounding an element or volume. Proportional to the area upon which the force acts.
  - Example: Friction along a fault plane

Surface stresses

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- **Surface stress**: A pair of equal and opposite forces acting on the area of a surface in a specific orientation

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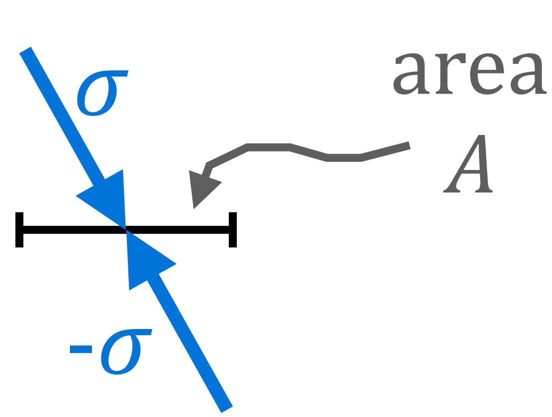
- **Example**: Hand pushing on table, table pushing back

Surface stress



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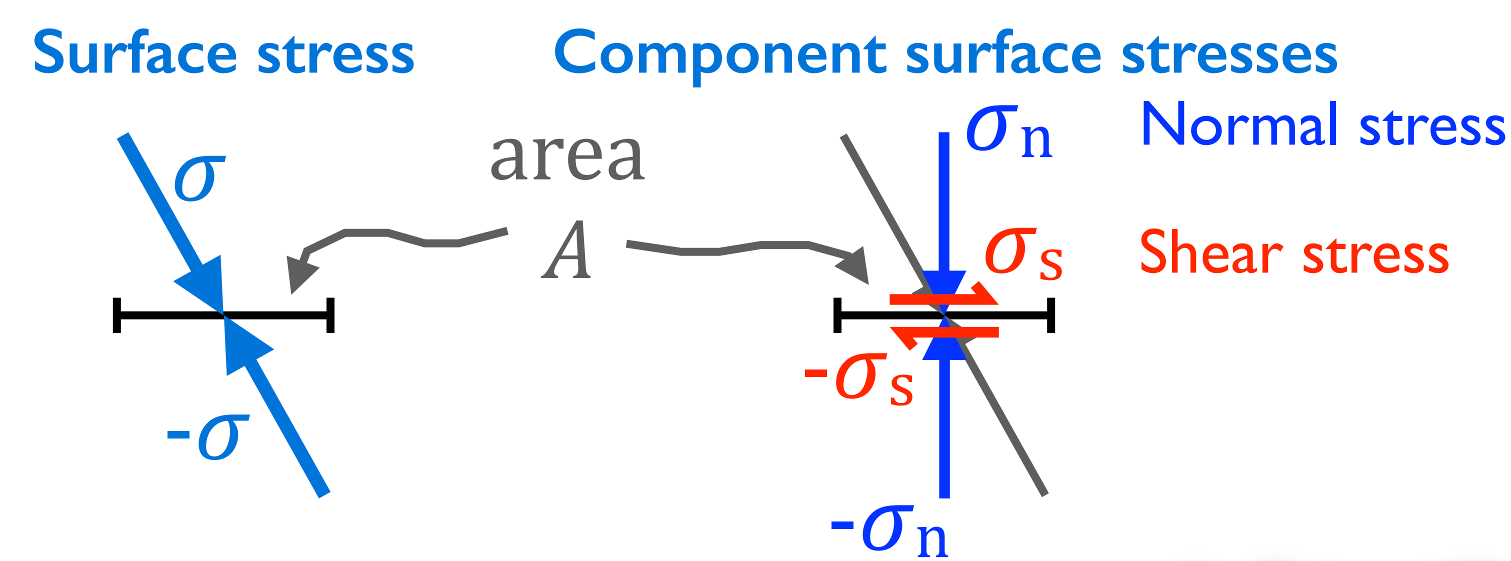

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Stress in two dimensions

Surface forces in 2D

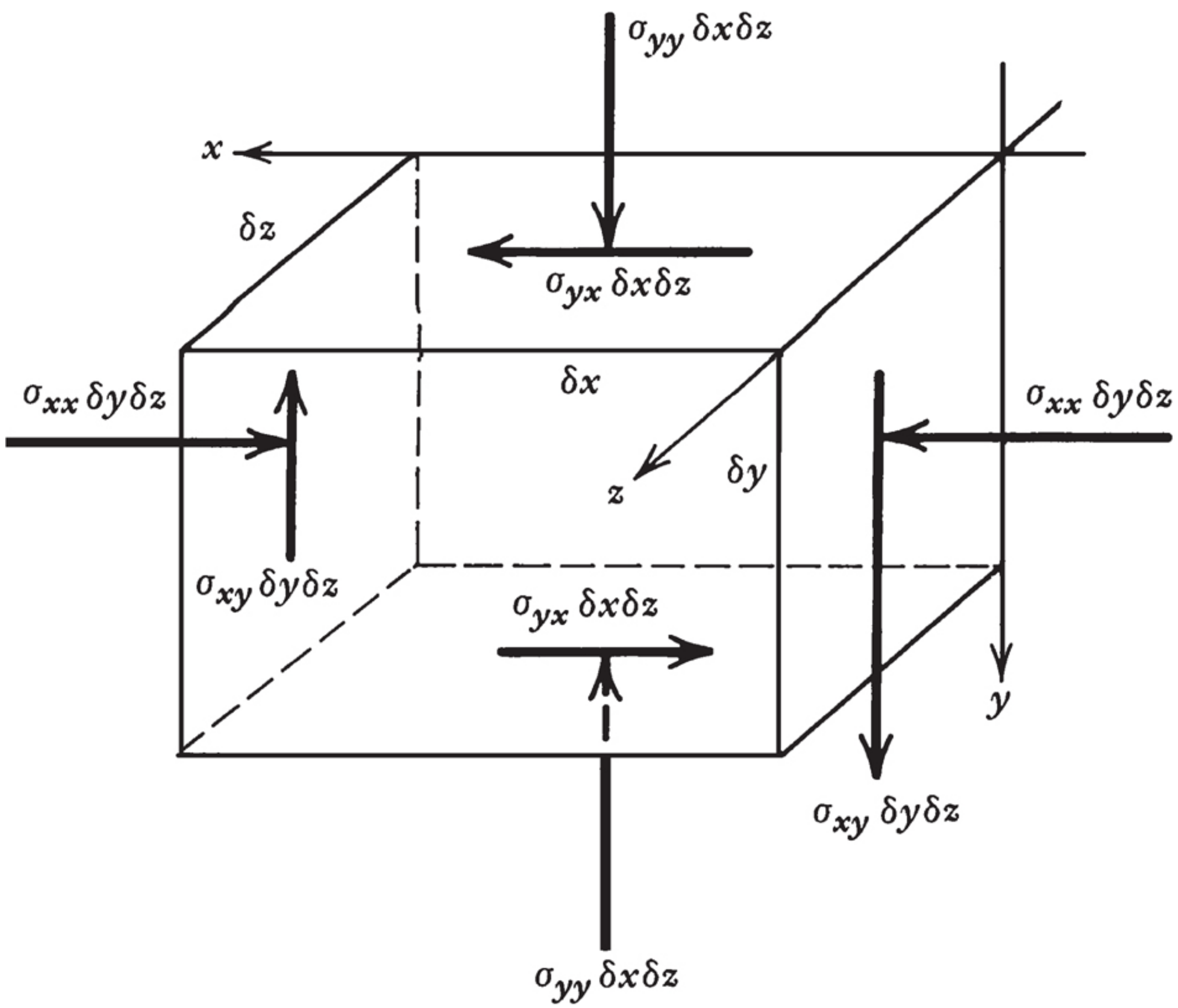
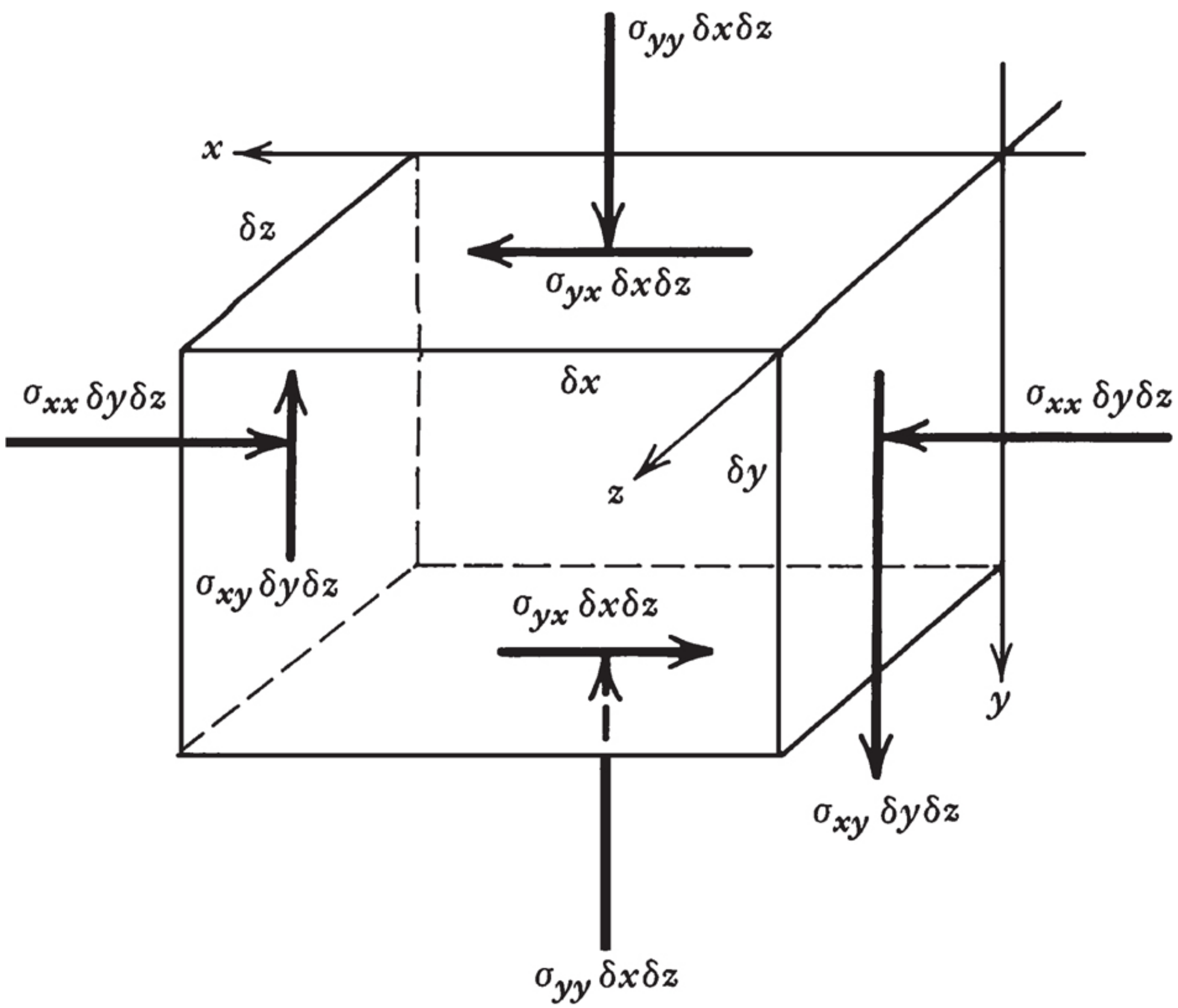


Fig. 2.13, Turcotte and Schubert, 2014

- In two dimensions, we consider forces acting on four faces of an infinitesimal cube of dimension $\delta x \times \delta y \times \delta z$
  - Here we assume no forces act or vary in the z direction

Stress in two dimensions

Surface forces in 2D



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- Shear stresses: σ_{xy}, σ_{yx}
- At equilibrium we can state $\sigma_{xy} = \sigma_{yx}$

Stress in two dimensions

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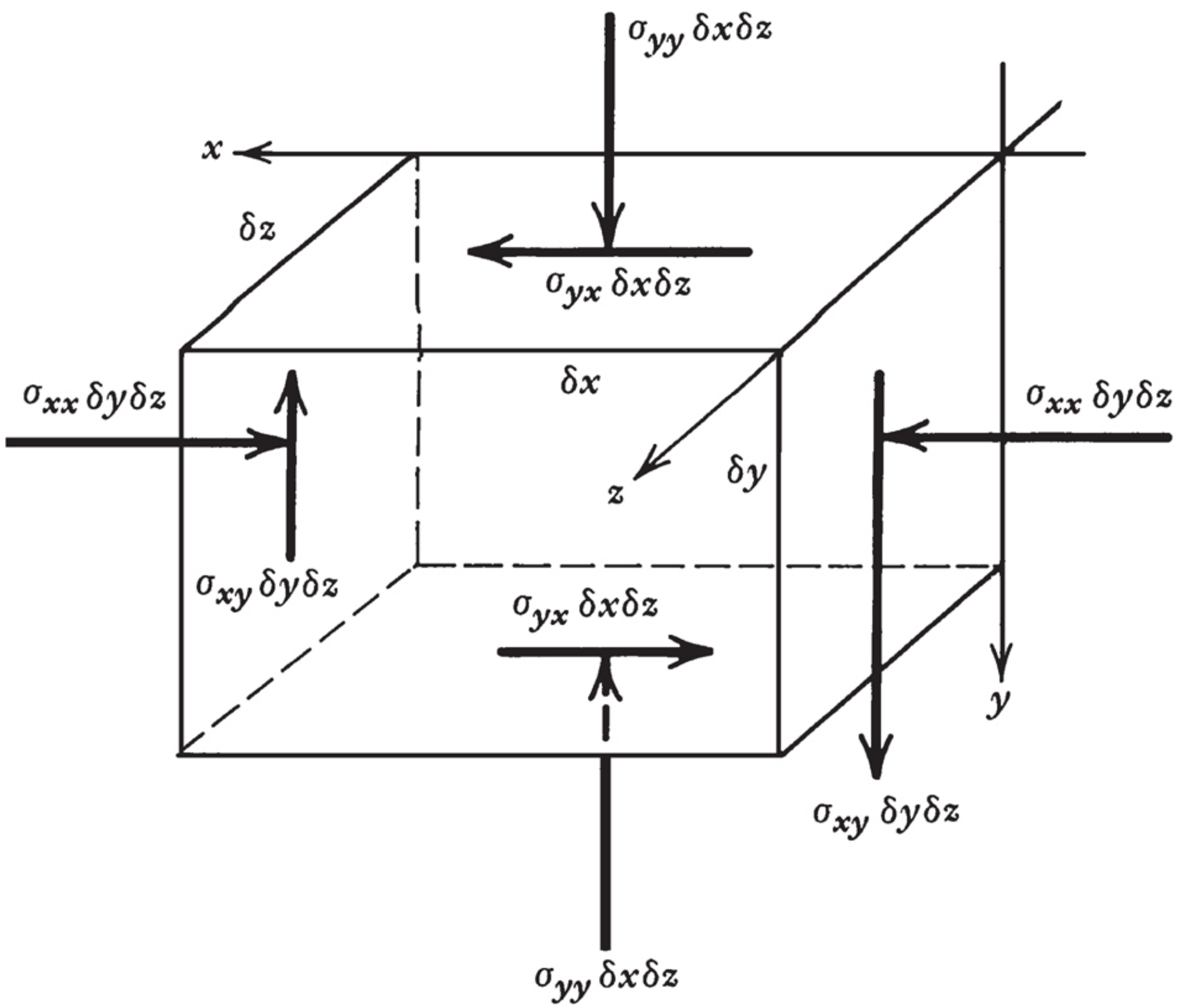

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  - Why?

Stress in three dimensions

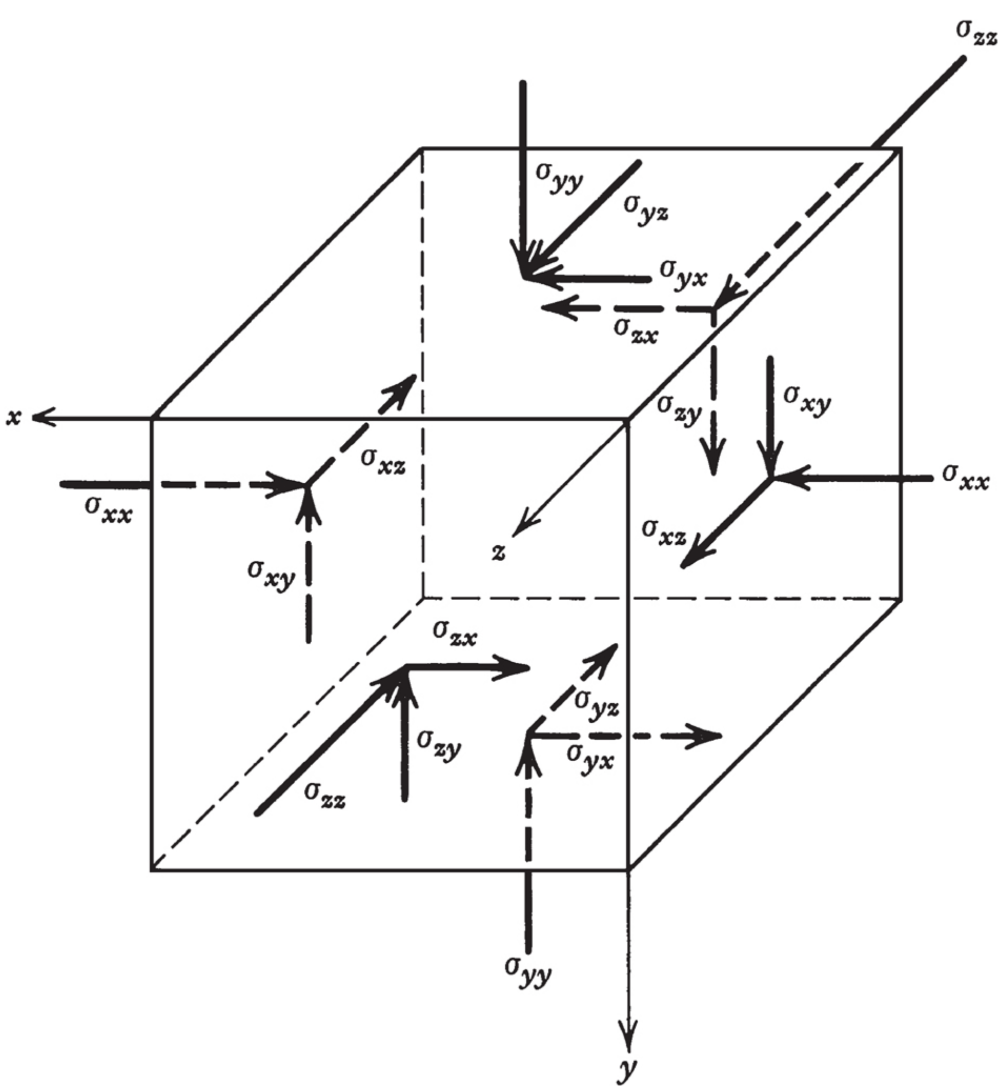


Fig. 2.15, Turcotte and Schubert, 2014

- In three dimensions, we consider forces acting on all six faces of an infinitesimal cube of dimension $\delta x \times \delta y \times \delta z$
- Normal stresses: σ_{xx}, σ_{yy}, σ_{zz}
- Shear stresses: σ_{xy}, σ_{yx}, σ_{xz}, σ_{zx}, σ_{yz}, σ_{zy}
- At equilibrium we can state $\sigma_{xy} = \sigma_{yx}$, $\sigma_{xz} = \sigma_{zx}$, $\sigma_{yz} = \sigma_{zy}$

Stress in three dimensions

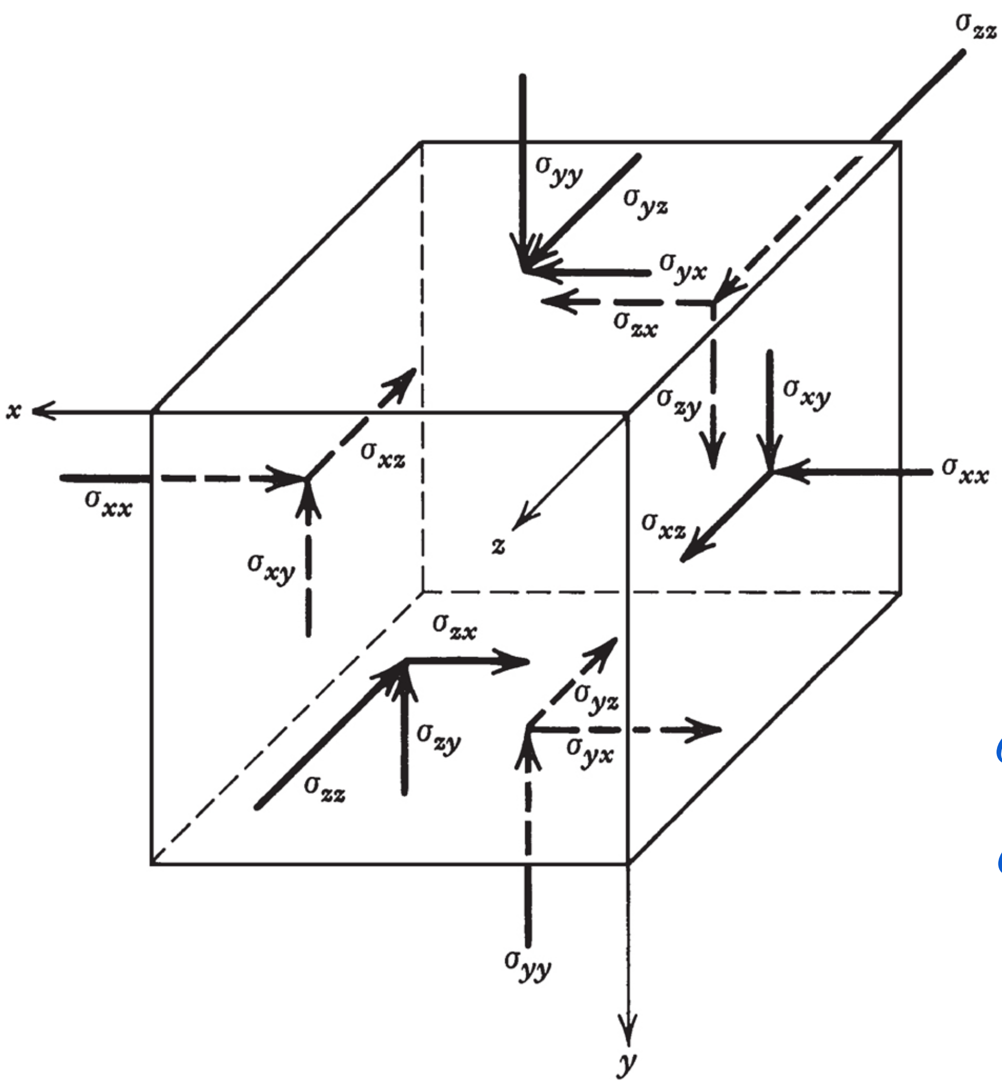


Fig. 2.15, Turcotte and Schubert, 2014

- A few useful stress values
  - **Pressure** (mean stress)

$$p = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

  - **Deviatoric stress** (indicated by primes)

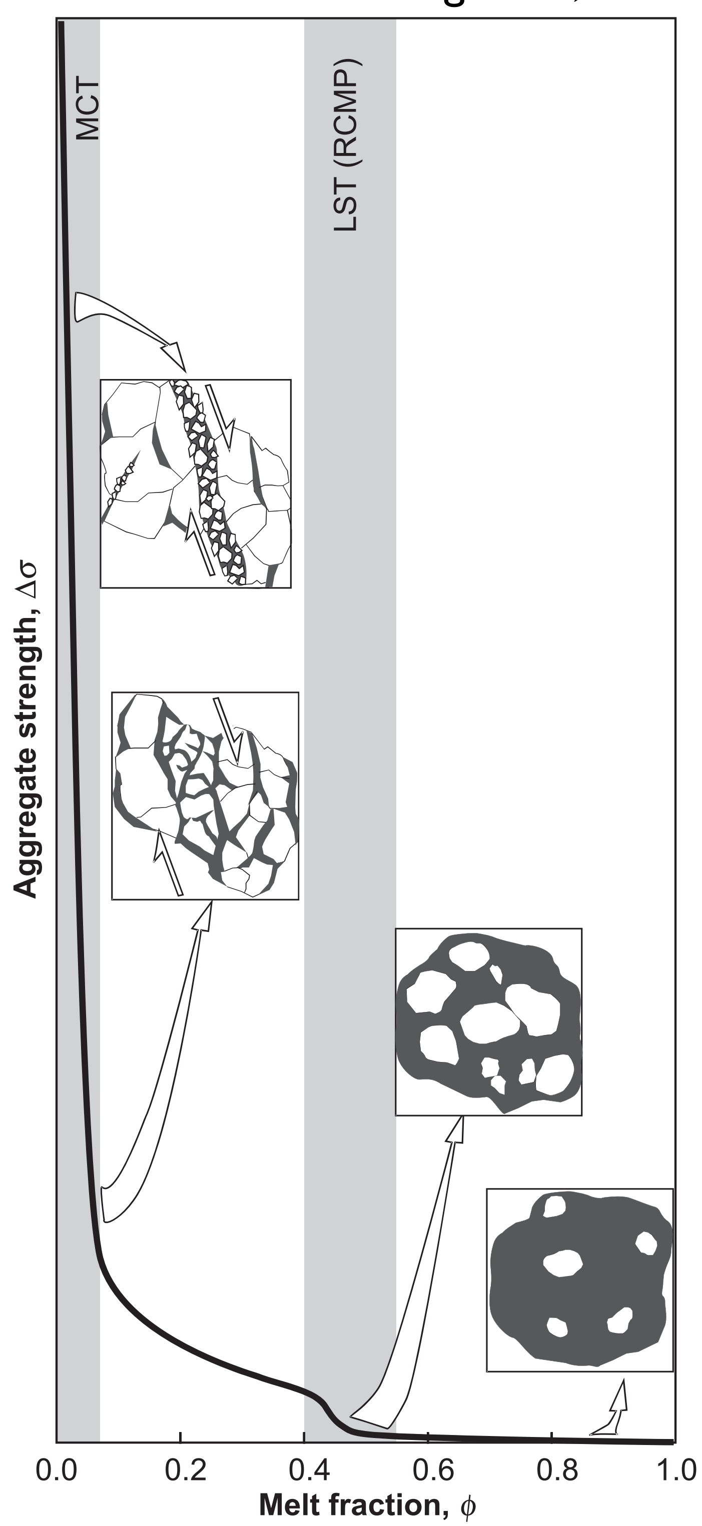
$$\sigma'_{xx} = \sigma_{xx} - p \qquad \sigma'_{yy} = \sigma_{yy} - p \qquad \sigma'_{zz} = \sigma_{zz} - p$$

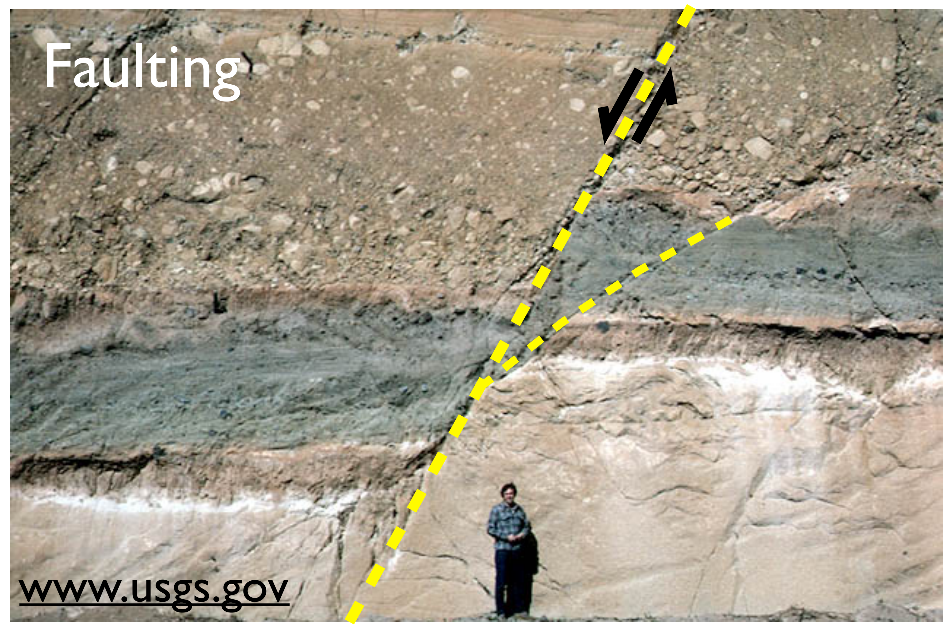
$$\sigma'_{xy} = \sigma_{xy} \qquad \sigma'_{xz} = \sigma_{xz} \qquad \sigma'_{yz} = \sigma_{yz}$$

Physical model concepts: Heat transfer

Why does temperature matter?

Rosenberg et al., 2007





- Rock deformation strongly depends upon temperature
  - Rock strength drops 80-90% for even small amounts of partial melt (5-7%)
  - Whether rocks are brittle and fault, or ductile and fold is largely determined by temperature

Heat transfer processes in the lithosphere

- **Conduction**

- **Production**

- **Advection**

Heat transfer processes in the lithosphere

- **Conduction**: The diffusive transfer of heat by kinetic atomic or molecular interactions within the material. Also known as thermal diffusion.

- **Production**

- **Advection**

Fourier's first law of heat conduction

- In 1D, the mathematical translation of "Heat flux q is *directly* proportional to the thermal gradient in a material" is

$$q_z = -k\frac{dT}{dz}$$

- Here, T represents **temperature** and z represents spatial position, **depth** in the Earth for our example
- Thus, dT/dz is the change in temperature with depth, or the **thermal gradient**
- The proportionality constant k is known as the **thermal conductivity**

Fourier's first law of heat conduction

- In 1D, the mathematical translation of "Heat flux q is *directly* proportional to the thermal gradient in a material" is

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- **Why is there a negative sign?**

Fourier's first law of heat conduction

- In 1D, the mathematical translation of "Heat flux q is *directly* proportional to the thermal gradient in a material" is

$$q_z = -k \frac{dT}{dz}$$

- **Why is there a negative sign?**

- In general, we can state Fourier's first law of heat conduction as

$$\boldsymbol{q} = -k \nabla T$$

Heat transfer processes in the lithosphere

- **Conduction**: The diffusive transfer of heat by kinetic atomic or molecular interactions within the material. Also known as thermal diffusion.

- **Production**: Not really a heat transfer process, but rather a source of heat. Sources in the lithosphere include radioactive decay, friction in deforming rock or chemical reactions such as phase transitions.

- **Advection**

Radiogenic heat production

- **Radiogenic heat production**, A or H, is one of several heat sources and results from the decay of radioactive isotopes in the Earth, mainly ^{238}U, ^{235}U, ^{232}Th and ^{40}K. A is often used for volumetric heat production and H for heat production by mass.

- These elements occur in the mantle, but are concentrated in the crust, where radiogenic heating can be significant
  - The surface heat flow in continental regions is ~65 mW m^{-2} and ~37 mW m^{-2} is from radiogenic heat production (57%)

Rock Type	U (ppm)	Concentration Th (ppm)	K (%)
Reference undepleted (fertile) mantle	0.031	0.124	0.031
"Depleted" peridotites	0.001	0.004	0.003
Tholeiitic basalt	0.07	0.19	0.088
Granite	4.7	20	4.2
Shale	3.7	12	2.7
Average continental crust	1.42	5.6	1.43
Chondritic meteorites	0.008	0.029	0.056

Turcotte and Schubert, 2014

Heat transfer processes in the lithosphere

- **Conduction**: The diffusive transfer of heat by kinetic atomic or molecular interactions within the material. Also known as thermal diffusion.

- **Production**: Not really a heat transfer process, but rather a source of heat. Sources in the lithosphere include radioactive decay, friction in deforming rock or chemical reactions such as phase transitions.

- **Advection**: The transfer of heat by physical movement of molecules or atoms within a material. A type of convection, mostly applied to heat transfer in solid materials.

Mathematical description of advection

[Time-dependent advection and diffusion]

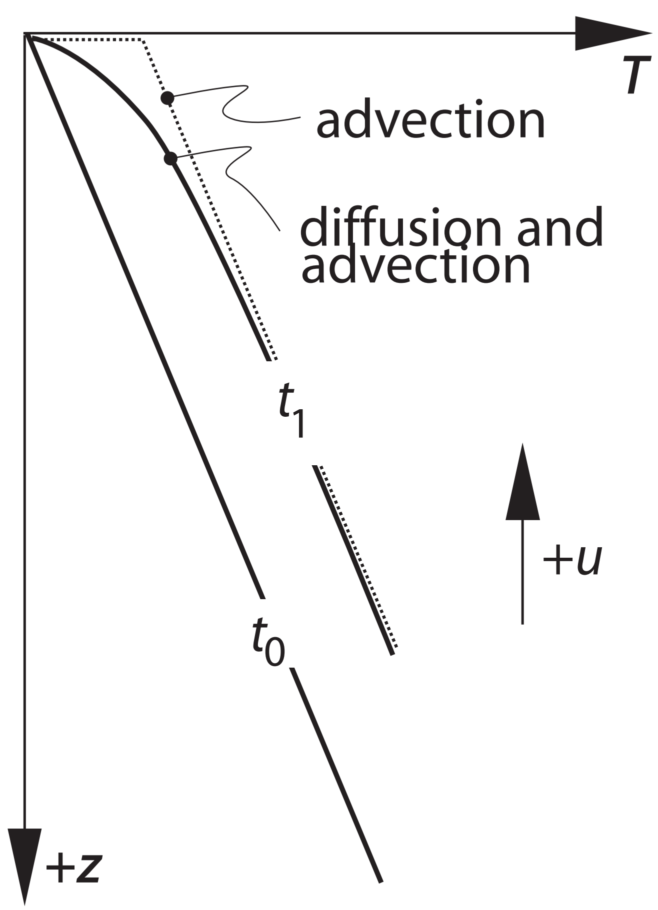

Fig. 3.13, Stüwe, 2007

- Advection <u>in the vertical direction at velocity</u> v_z *at steady state* can be represented mathematically as

$$v_z \frac{dT}{dz} = 0$$

  - Note that this equation simply describes the vertical translation of temperatures, and that in order for any change in temperature to occur, advection must be combined with other heat transfer processes such as conduction

- In general, we can describe head advection as

$$\boldsymbol{V} \cdot \nabla T = 0$$

The heat conservation equation

- We now combine our three heat transfer components (**conduction**, **production**, **advection**) into the heat conservation equation, which describes heat transfer subject to each of these processes

- In one dimension (vertical), this equation is

$$\rho c_P \left(\frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} \right) = -\frac{\partial q_z}{\partial z} + A$$

Time dependence, Advection, Conduction, Production

- Alternatively, we can state the same equation in substituting in Fourier's first law for the heat flux q

$$\rho c_P \left(\frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + A$$

The heat conservation equation

- We now combine our three heat transfer components (**conduction**, **production**, **advection**) into the heat conservation equation, which describes heat transfer subject to each of these processes
- In general, we can state

$$\rho c_P \left(\frac{\partial T}{\partial t} + \boldsymbol{V} \cdot \nabla T \right) = \nabla \cdot k \nabla T + A$$

Time dependence Advection Conduction Production

Physical model concepts: Rheological laws

Rheology of the lithosphere

- The term **rheology** refers to the flow characteristics of materials
  - For most geoscientists this term describes the deformation behavior of materials regardless of whether deformation occurs by flow, fracture, or other mechanisms
- Rock deformation mainly occurs by three **deformation mechanisms**:
  - Elasticity
  - Plasticity
  - Viscous flow

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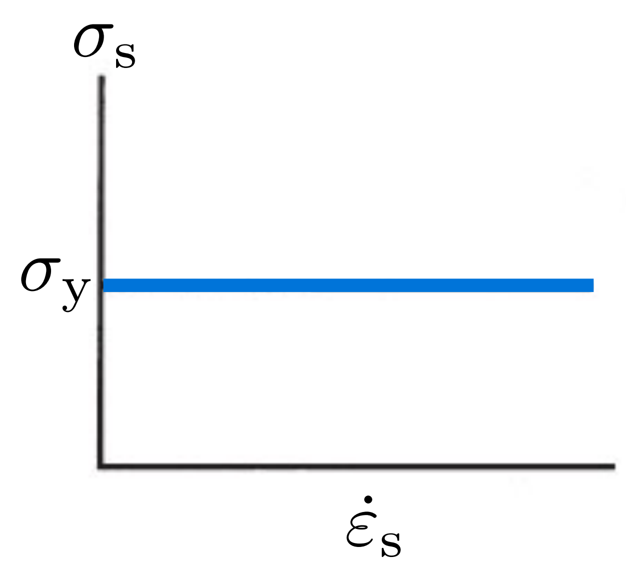
    - For most geoscientists this term describes the deformation behavior of materials regardless of whether deformation occurs by flow, fracture, or other mechanisms

- Rock deformation mainly occurs by three **deformation mechanisms**:
    - ~~Elasticity~~ - Can ignore this, not relevant for long time scales
    - Plasticity
    - Viscous flow

Perfectly plastic behavior

Twiss and Moores, 2007

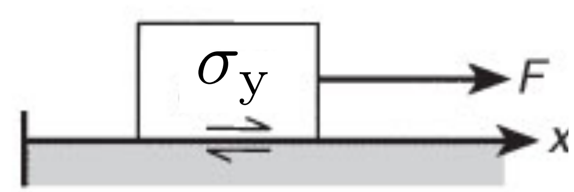


A.

- **Constant stress** required for deformation
  - No deformation prior to exceeding yield stress
  - Infinite deformation if applied stress equals (or exceeds) yield stress

$$\begin{cases} \sigma < \sigma_y & \text{no deformation} \\ \sigma = \sigma_y & \text{failure; infinite deformation} \end{cases}$$

- Nonrecoverable



B.

Perfectly plastic behavior

Twiss and Moores, 2007

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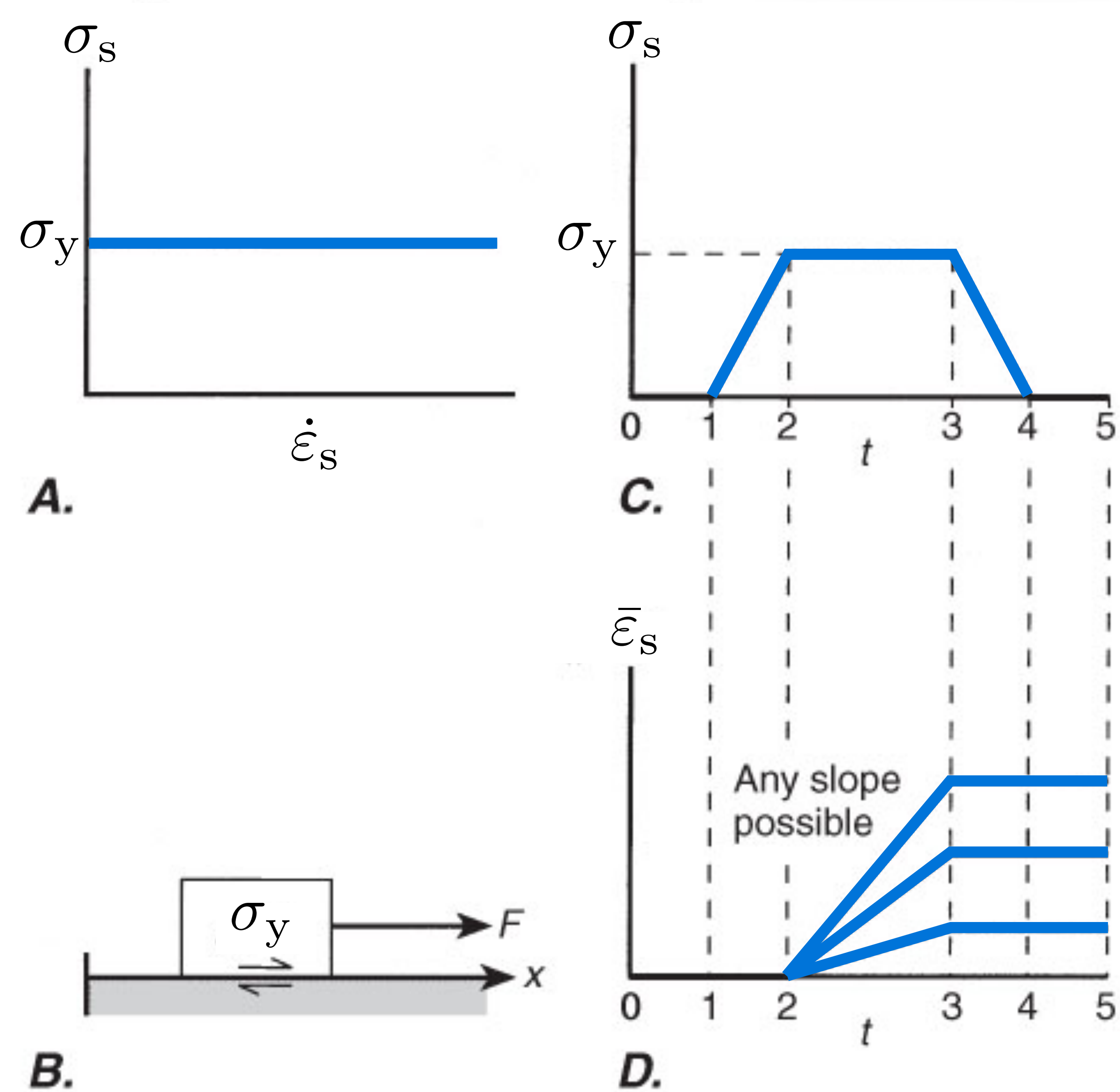
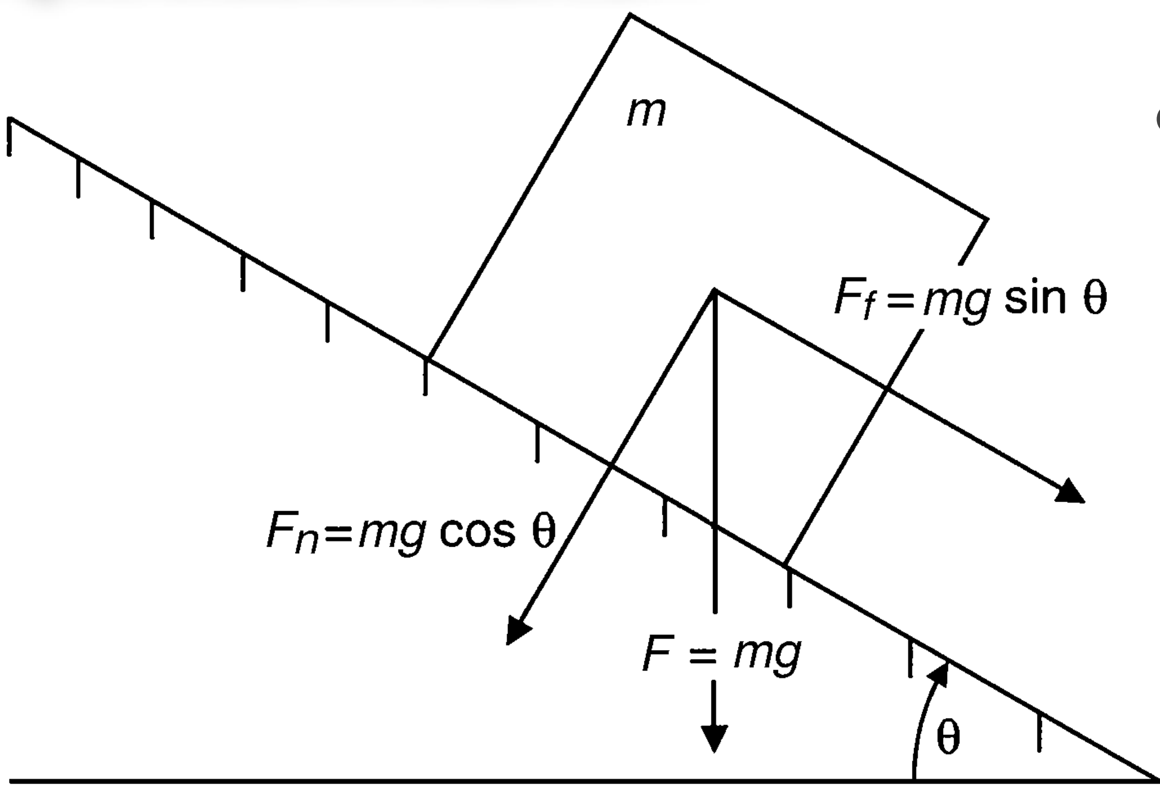


Frictional plasticity

Fig. 8.5, Turcotte and Schubert, 2014



Normal stress $\sigma_{\rm n} = \dfrac{mg\cos\theta}{A}$

Shear stress $\sigma_{\rm s} = \dfrac{mg\sin\theta}{A}$

- Fault slip accounts for a large portion of deformation of the upper crust
- Friction must be overcome for slip to occur
  - After exceeding the frictional resistance, slip will occur on the fault or shear zone
  - Known as **frictional plasticity**
  - The basic relationship for static friction is

$$\tau_{f_{\rm s}} = f_{\rm s}\sigma_{\rm n} \qquad \text{(Amonton's law)}$$

where f_s is the **coefficient of static friction**, and τ_{fs} is the **static frictional stress** required for slip

(Linear) Viscous deformation

- In simple shear,

  $\tau_s = \eta\dot{\gamma}$ η Dynamic viscosity

  Shear stress proportional to **shear strain rate**

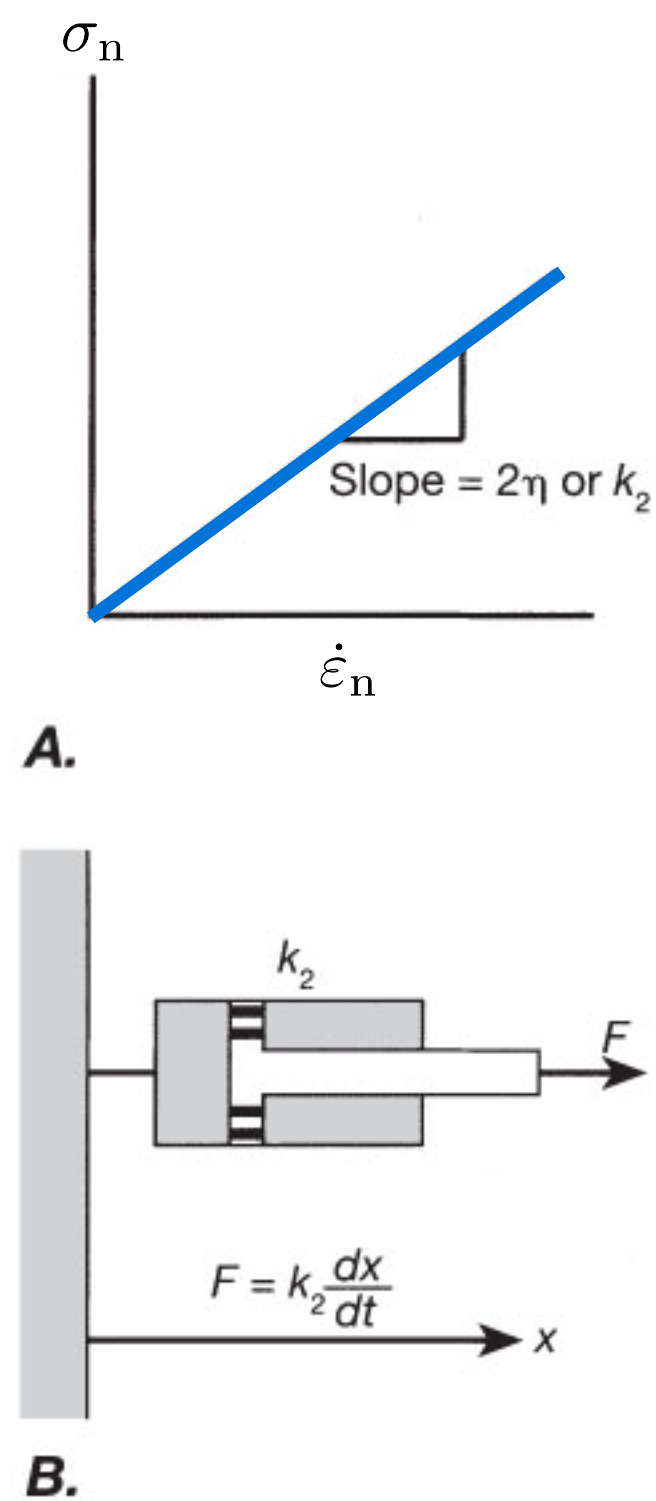
- In general,

  $\boldsymbol{\tau} = 2\eta\dot{\boldsymbol{\varepsilon}}$

  deviatoric stress is proportional to **strain rate**

- For linear viscous (Newtonian) materials, η is constant

- Nonrecoverable



Twiss and Moores, 2007

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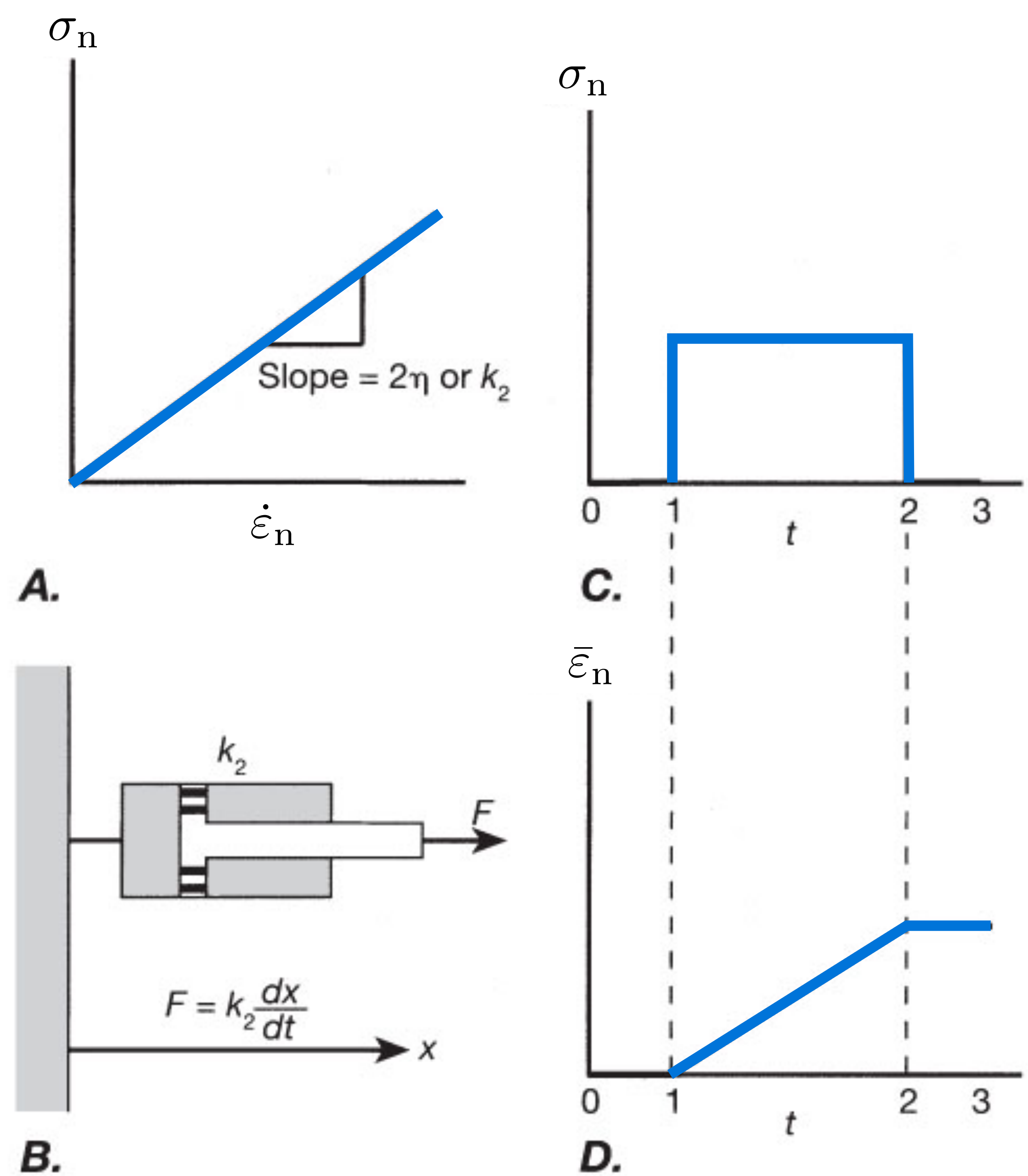


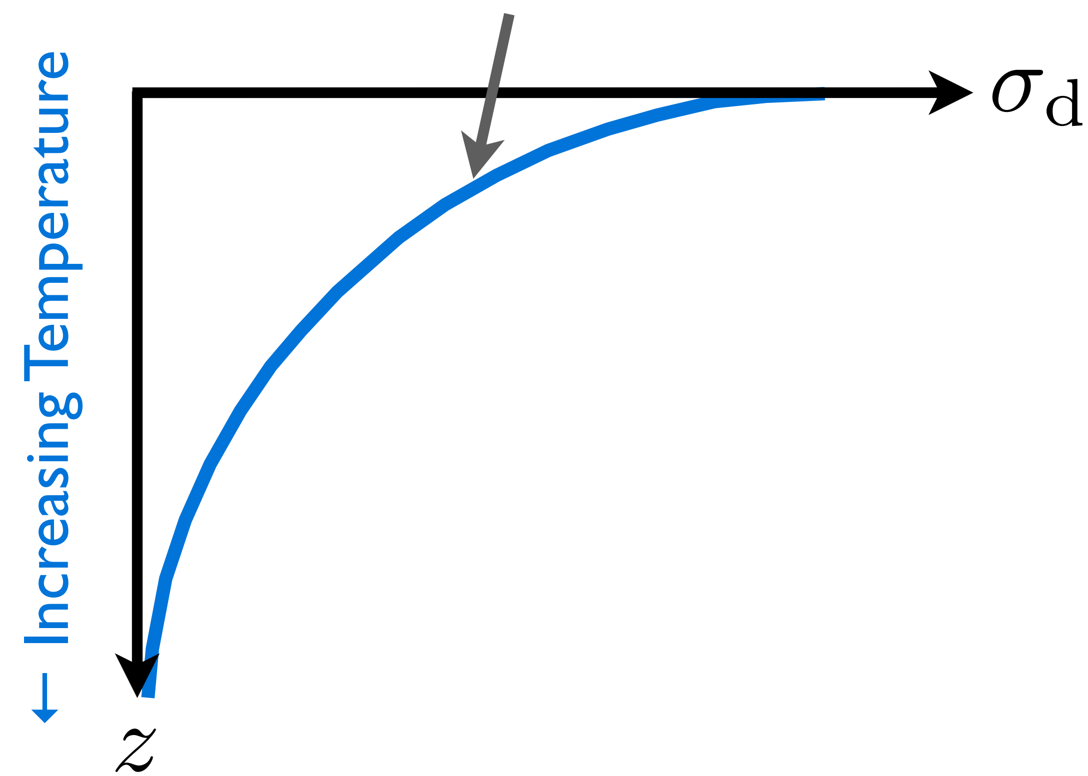
Twiss and Moores, 2007

Nonlinear viscous deformation

- Most rocks do not behave as Newtonian viscous materials
- Why not?
- Two main reasons:
    - Temperature dependence

$$\eta = A_0 e^{Q/RT_\mathrm{K}}$$

A_0 is the pre-exponent constant, Q is the activation energy, R is the universal gas constant and T_K is temperature in Kelvins



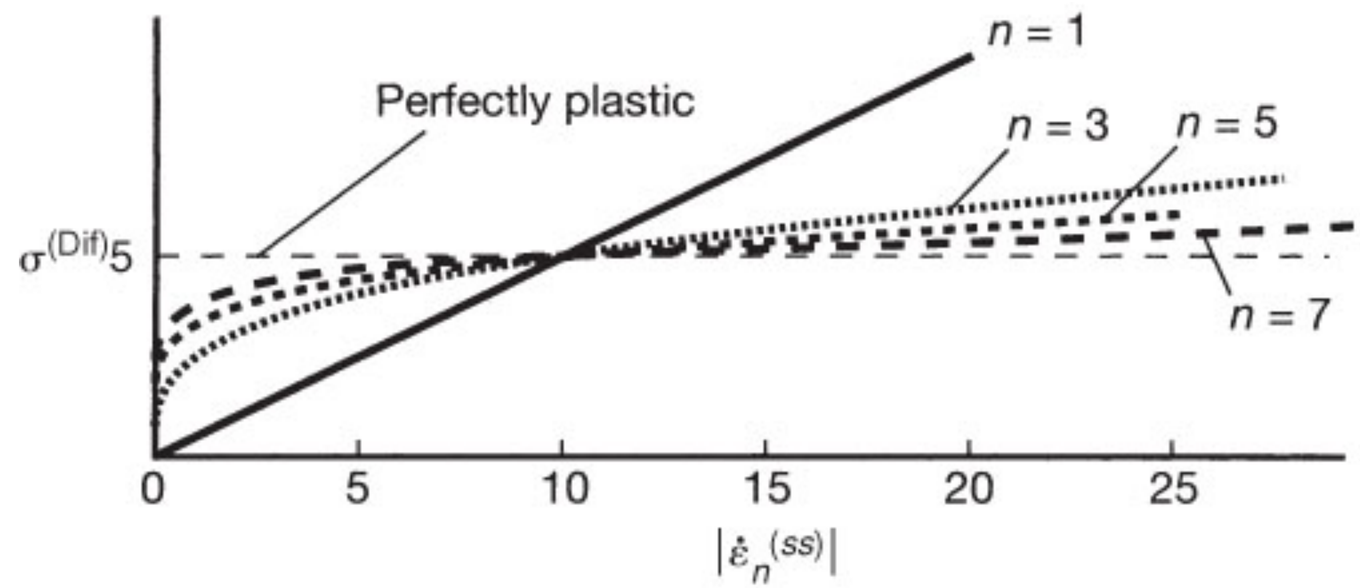
Stüwe, 2007

Nonlinear viscous deformation

- Most rocks do not behave as Newtonian viscous materials
- Why not?
- Two main reasons:
  - Nonlinearity

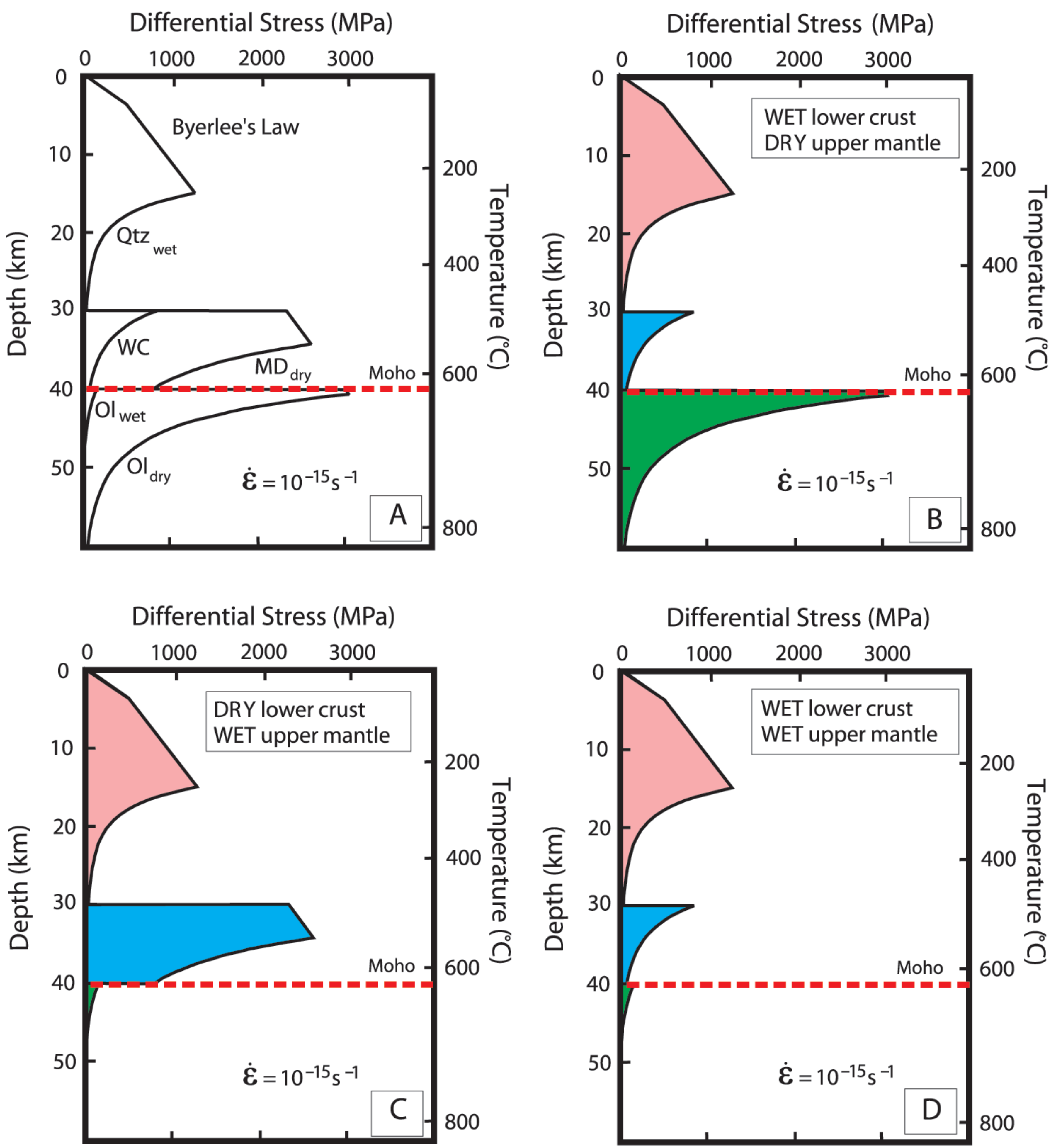
    $$\tau_s^n = A_{\text{eff}}\dot{\gamma}$$

    n is the power law exponent and A_{eff} is a material constant in $\text{Pa}^n\,\text{s}$
  - Many rocks deform 8 times as fast when stress is doubled



Twiss and Moores, 2007

Lithospheric strength envelopes



- There are many ways in which lithospheric strength can be modelled, here are a few
- **Jelly sandwich**
  - A - Brace-Goetze
  - B - Wet LC
- **Crème brûlée**
  - C - Wet UM
  - D - Wet LC, UM

Jackson, 2002

Summary I

- The aim of this course is to help you understand lithospheric geodynamic models
  - The models are (thermo-)mechanical, where the internal and external forces acting on the extremely viscous fluid in the model determine how the model will deform
  - The physics and general concepts of the equations are fairly simple, but as you will see, the numerical solution of the equations and the output can be complex

Summary II

- Deformation of the Earth in numerical geodynamic models is based on three simple conservation equations
  - **Conservation of mass** - The continuity equation
  - **Conservation of momentum** - The momentum equation
  - **Conservation of energy** - The heat transfer equation

- Conservation of mass, momentum and energy are combined with rheological laws to describe fluid movement under an applied force

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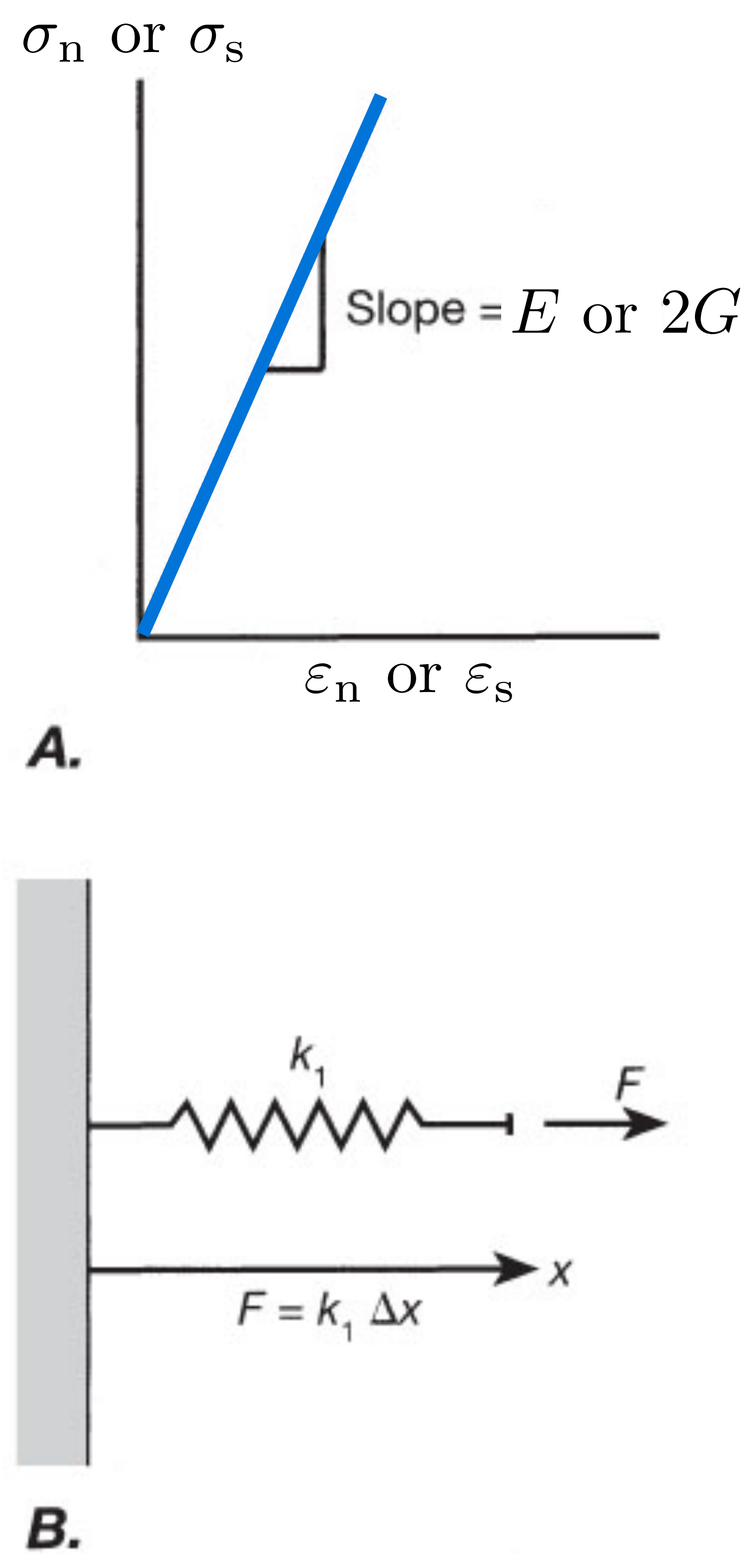
Extra slides

Elasticity

Twiss and Moores, 2007



$$\sigma \propto \varepsilon$$

- **Stress** is proportional to **strain**
- For 1-D normal stress

$$\sigma_{xx} = E\varepsilon_{xx}$$

E : Young's modulus (1D)

G : Shear modulus (1D)

- If stress → 0, strain → 0 (recoverable)

Elasticity

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Twiss and Moores, 2007