Introduction to geodynamic modelling

Introduction to DOUAR

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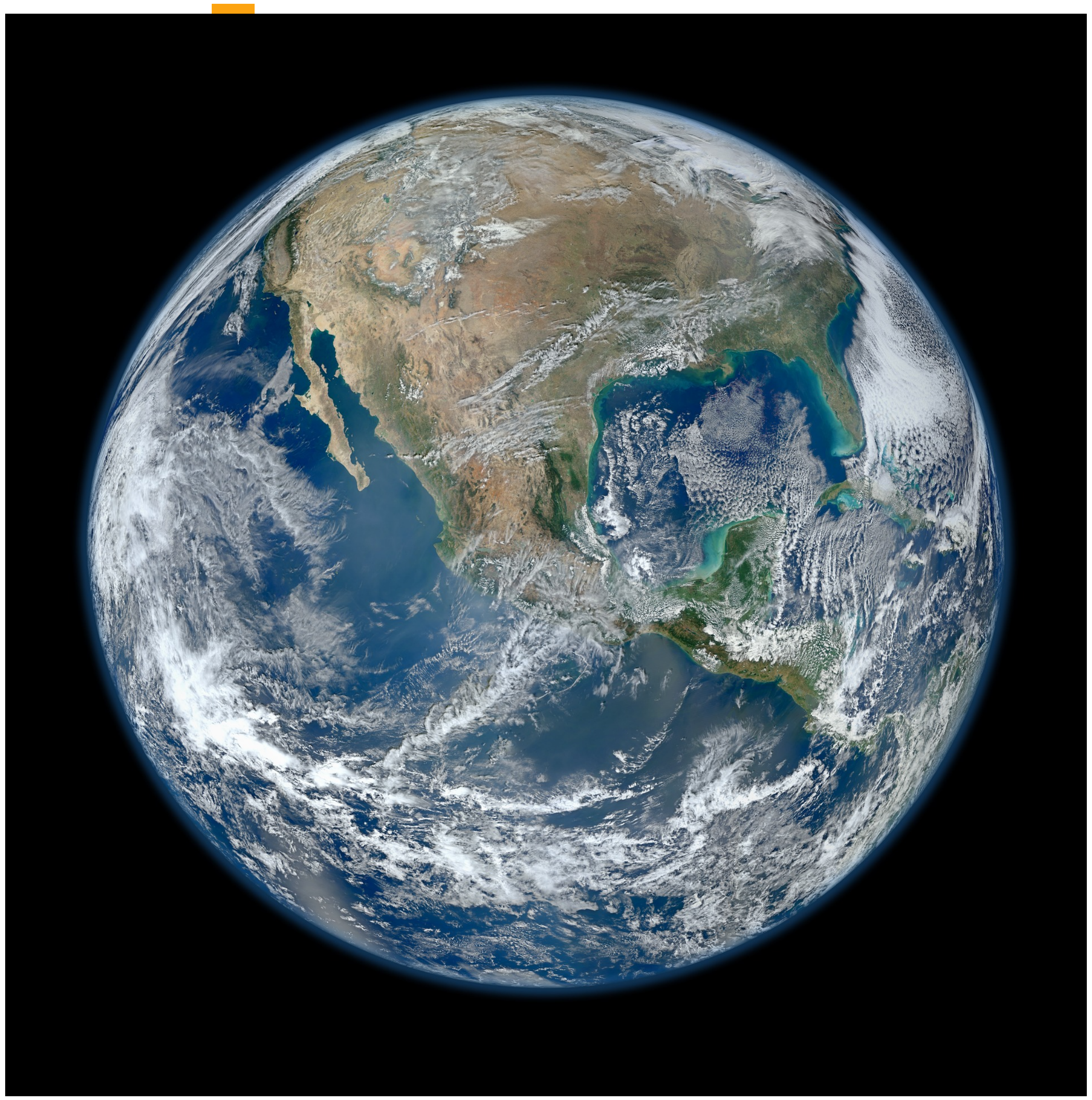
Goals of this lecture

- Introduce **DOUAR**, a 3D thermomechanical numerical modelling program for creeping flows

- Present some of the **important features** in DOUAR that will be relevant for our use

DOUAR in a nutshell



DOUAR is the word for Earth in the Breton language

- **DOUAR** (Braun et al., 2008) is a 3D finite-element code for modelling geodynamic processes
- It is designed to be run efficiently in parallel on computer clusters to be able to solve 3D problems at high spatial resolution
- We don’t have time for a complete description of the numerical and computational aspects of DOUAR, but we’ll see the highlights

Physics of DOUAR

- DOUAR calculates the flow of a highly viscous flow using the Stokes equation, which we have previously seen

$$\nabla \cdot \eta(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^{\mathrm{T}}) - \nabla p = \rho g$$

where η is the fluid viscosity, $\boldsymbol{v}$ is the fluid velocity, p is pressure, ρ is density, and g is the acceleration due to gravity.

Incompressible flow, almost

- It is further assumed that the fluid is nearly incompressible
  - In an incompressible fluid the divergence of the velocity field is zero

$$\nabla \cdot \boldsymbol{v} = 0$$

  - In DOUAR, the slight compressibility of the fluid is used to determine fluid pressure (eliminating it from the Stokes equation) using the penalty method

$$-\lambda \nabla \cdot \boldsymbol{v} = p$$

where λ is the penalty factor (typically 8 orders of magnitude larger than the shear viscosity)

Rheologies

- Materials in DOUAR are either viscous or plastic (no elasticity)
  - Nonlinear viscosity is modelled using the equation for temperature-dependent nonlinear viscosity

$$\eta = \eta_0 \dot{\varepsilon}^{1/(n-1)} \exp\left(Q/nRT\right)$$

where η_0 is the viscosity pre-factor, $\dot{\varepsilon}$ is the strain rate, n is the power law exponent, Q is the activation energy, R is the universal gas constant, and T is temperature in Kelvins

Rheologies

- There are several options for different plasticity criteria
  - The Mohr-Coulomb criterion is what we use most often

$$\tau = c - \sigma_{\mathrm{n}} \tan \phi$$

where τ is the shear stress, c is the cohesion, σ_{n} is the normal stress, and ϕ is the internal angle of friction

Thermal model

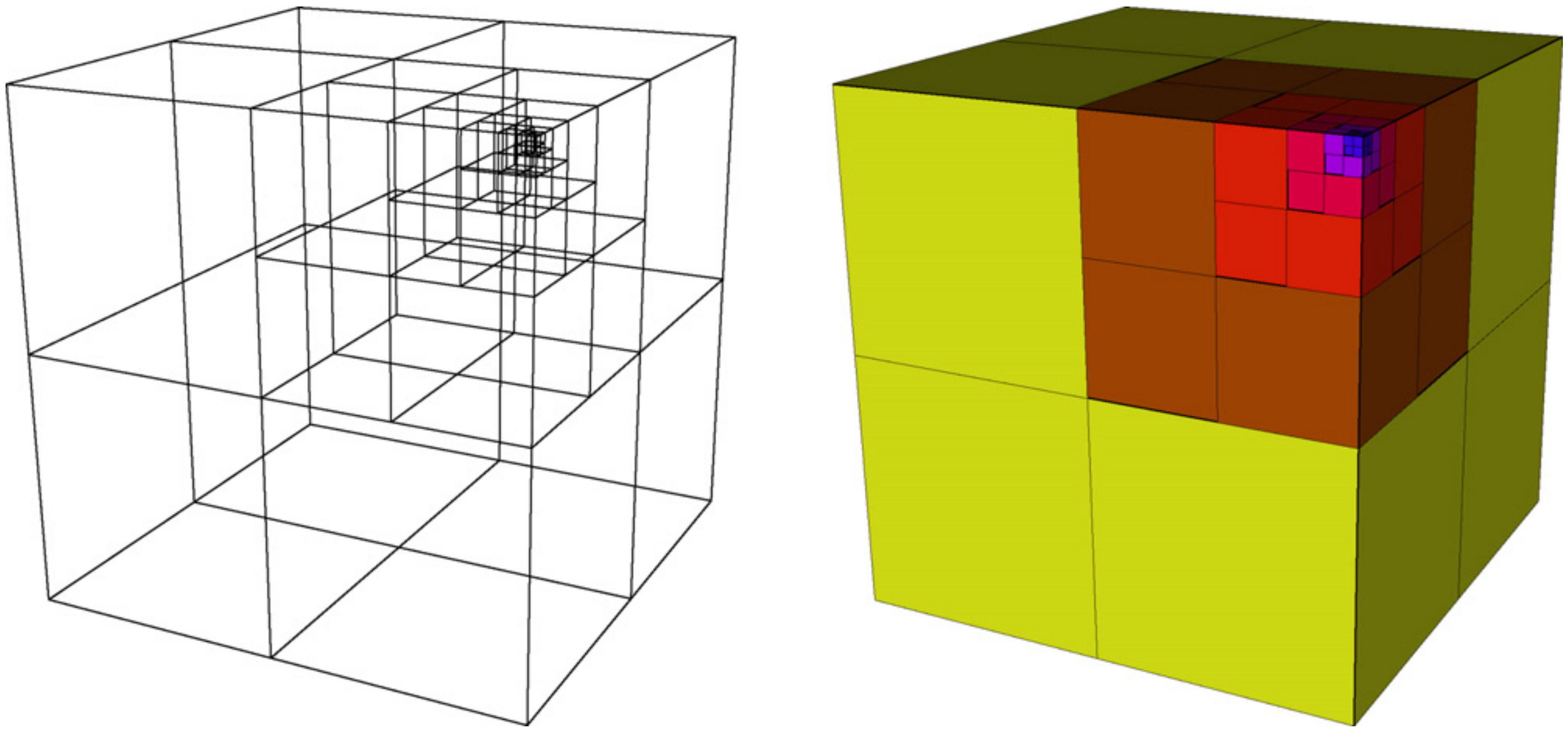
- The full 3D advection-diffusion equation with heat production is solved in DOUAR

$$\rho c \left(\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T \right) = \nabla \cdot k \nabla T + \rho H$$

where c is the heat capacity, T is temperature, t is time, k is the thermal conductivity, and H is the heat production per unit mass.

Numerical approach

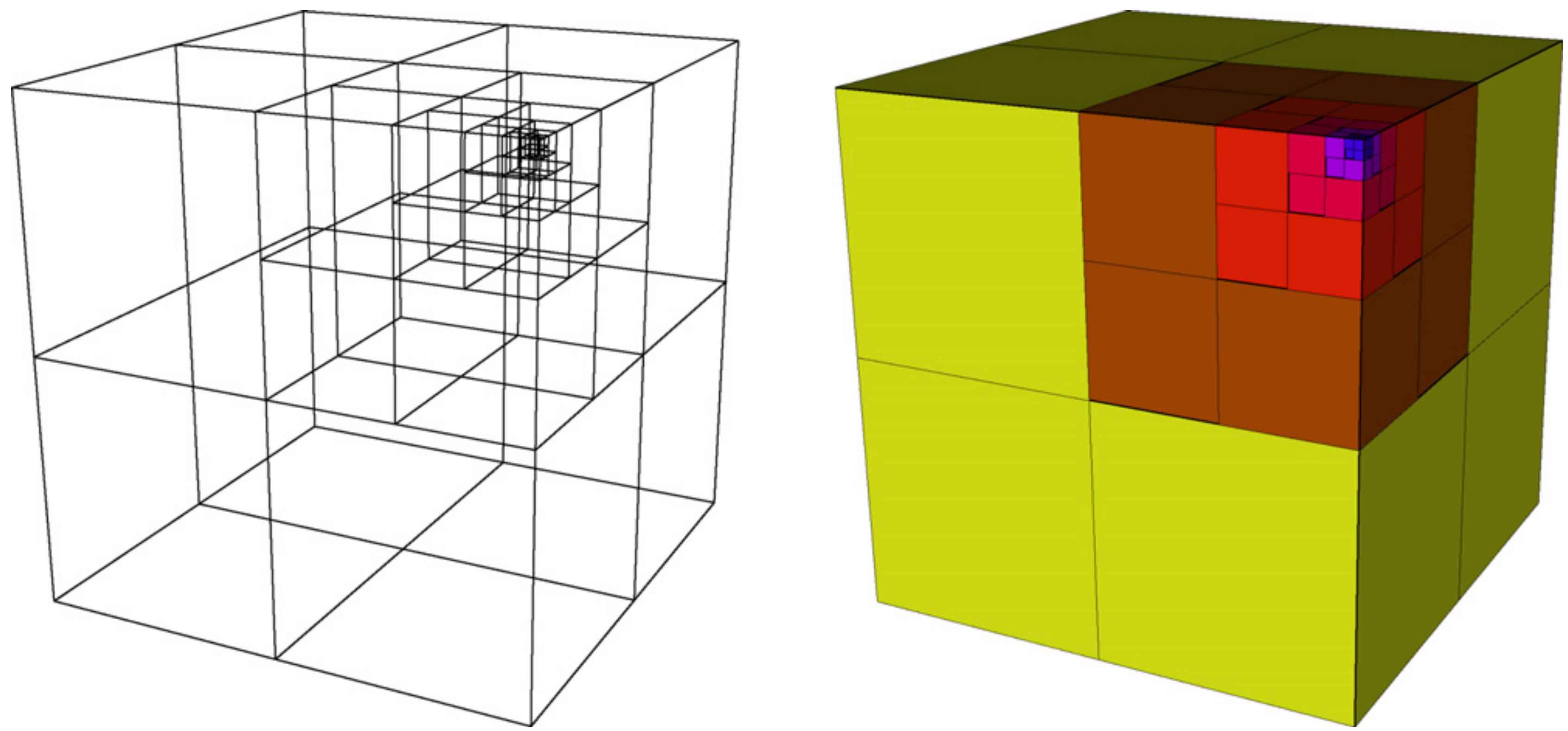
Braun et al., 2008



- The finite-element mesh used in DOUAR is Eulerian and based on the octree division of space
  - An Eulerian mesh is one that is fixed in space with respect to the fluid flowing within/through it
  - The octree division of space is based on subdivisions of a unit cube

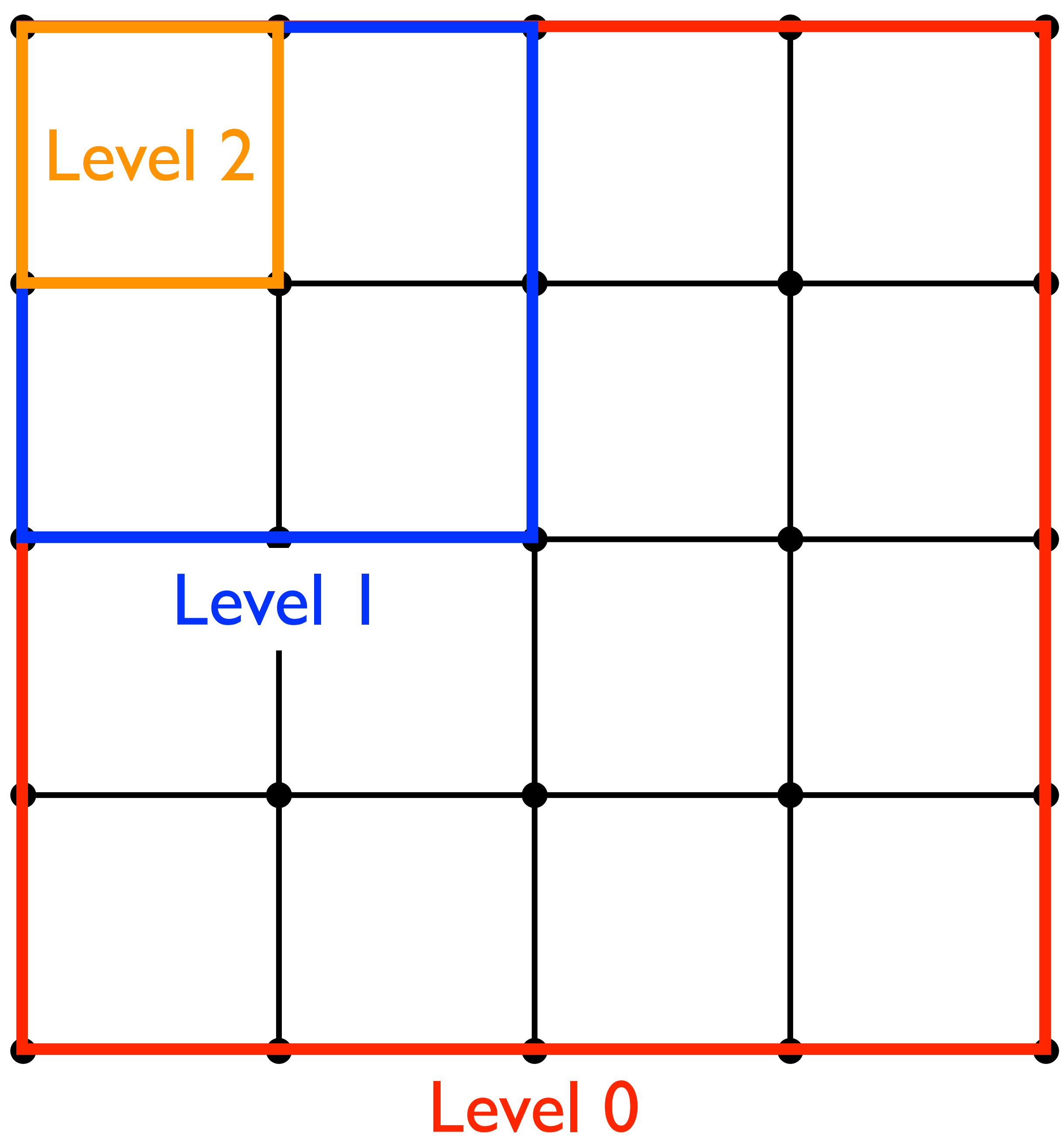
Numerical approach

Braun et al., 2008



- The unit cube is at the octree level 0 resolution, meaning that there are 2^0 elements along the width of the cube
- A mesh at octree level 6 would have 2^6, or 64 x 64 x 64 elements
- The initial octree resolution is an important setting in the input file

The octree division of space



- As an example, here is a face of a DOUAR model (2D, not full cube) at octree level 2
  - This is 2 subdivisions of the unit cube (red)

Other aspects of DOUAR

- (continued on whiteboard)

References

Braun, J., Thieulot, C., Fullsack, P., DeKool, M., Beaumont, C. and Huismans, R., 2008. DOUAR: A new three-dimensional creeping flow numerical model for the solution of geological problems. *Physics of the Earth and Planetary Interiors*, *171*(1-4), pp.76-91.